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DYNAMIC MODEL STUDY OF LAKE ERIE

Part II. Analytical and Experimental Results

by

Ralph R. Rumer, Jr.

August 1970

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27 Abstract
This second report considers the experimental results obtained and the relationship of these results to various mathematical model approaches to the hydraulic study of Lake Erie. Based on the experimental and analytical results of this study, the following summarizing statements are made regarding the dynamic behavior of Lake Erie.

In the absence of wind, the throughflow caused by the inflow of the Detroit River and the outflow of the Niagara River generates significant currents and easily observable patterns of circulation.

The presence of westerly wind significantly alters the circulation pattern of Lake Erie as observed in the absence of wind. A procedure for relating model wind speeds to prototype wind speeds based on similarity between wind set-up in model and prototype has been presented. The period for the first mode mass oscillation in the model lake is in good agreement with calculated and observed periods. Observed detention periods show significant short circuiting for the homogeneous lake with no wind stress. Only limited correspondence can be achieved in modeling dispersion in vertically distorted Froude models of very large bodies of water. (See also W71-00621)

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DYNAMIC MODEL STUDY OF LAKE ERIE

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Notation

from Part I

C_D = coefficient of drag
D = depth of frictional influence
E_{xy} = mass transport coefficient in horizontal plane
E_z = mass transport coefficient in vertical direction
F = Froude number
g = acceleration of gravity
H = average depth of lake
L_o = reference length
m = subscript denoting model quantity
n = Manning roughness coefficient
p = pressure, subscript denoting prototype quantity
$\bar{q}$ = velocity vector
r = subscript denoting ratio of model quantity to prototype quantity, radial distance measured from center of rotation of model
R_o = Rossby number
R^t = turbulent Reynolds number
S^t = wind set-up
t = time
U_o = reference velocity
V = average wind speed
x,y,z = coordinate directions
y_o = distance from water surface at center of rotation to horizontal plane
ϵ, ϵ_z = kinematic eddy viscosity
ρ = fluid density
ν = kinematic molecular viscosity
τ_s = wind shear on lake surface
ϕ = latitude
ω = rotation vector of lake
Ω = angular velocity of earth
∇ = vector operator
∇^2 = scalar operator

from Part II

A = cross sectional area of basin
A,C,D,E,F = coefficients
B = top width of basin
C_o, C_1 = coefficients
D = coefficient
E_x, E_y, E_z^n = dispersion coefficients
f = Coriolis parameter, friction coefficient
g,G = arbitrary functions of ψ
k = bottom roughness height
ℓ = basin length
n = number of terms in series expansion
T = seiche period
U_m = maximum seiche period
U,V^m = depth integrated velocity components

α = decay modulus coefficient
η = free surface displacement
τ_o = bottom shear stress
τ_x, τ_y = wind shear stresses
ψ = stream function

FOREWARD

This is the second of a two-part report dealing with a hydraulic model study of Lake Erie. The first part (Part I. Similitude Criteria and Experimental Setup, Rumer (1970)) considered the theoretical justification for the operation of a rotating vertically-distorted Froude model. It also described the experimental facilities designed and constructed for this study as well as the experimental procedures used in the acquisition of experimental data.

This second report considers the experimental results obtained and the relationship of these results to various mathematical model approaches to the hydraulic study of Lake Erie. Prototype observations, where available, are also discussed. This second report ends by making recommendations for continued investigations in which a hydraulic model can be an important adjunct to the overall study of lake circulation and water quality management.

VI LAKE CIRCULATION

VI.1 Introduction

The most fundamental cause for water movement in the Lake Erie basin is the throughflow generated by the inputs from the various tributary discharges into the lake and the single outflow through the Niagara River. The net water mass exchanged each year in Lake Erie by this throughflow amounts to more than thirty per cent of the total lake volume. Superimposed upon this throughflow is the wind-driven circulation and the seiche-induced circulation. Both of these secondary modes of water movement are significant and under certain conditions they can dominate the structure of the currents in the lake such that the throughflow pattern is barely distinguishable. Other factors which have a significant effect on how the water mass responds to the throughflow, wind shear, and seiching are the seasonal occurences of thermal stratification and ice coverage.

The first objective of the present study was to determine the nature of the steady-state circulation pattern for the throughflow for an isothermal lake condition. Since all of the discharges of the tributary rivers compared to that of the Detroit River inflow are very small (the Maumee River flow of 4794 cfs is the second largest compared to the Detroit River inflow of approximately 190,000 cfs), they were not included in the initial studies.

The simulation of the circulation pattern for an isothermal lake and in the absence of wind shear is a logical first step in gaining an understanding of the dynamic behavior of Lake Erie. This condition was studied in both the vertically distorted model of Lake Erie and the idealized rectangular basin of constant depth. Two mathematical models

were investigated in anticipation that the results of these analytical studies would assist in the interpretation of the experimental results. Comparison of the results for this first study with prototype observations is very limited since, in nature, the wind is almost ever present. Nevertheless, the posed condition of zero wind shear is approximated in the prototype when the lake is nearly covered by ice or after several days of calm weather.

The second objective of the present study was to discover the changes in the general circulation pattern due to the application of a steady uniform westerly wind over the surface of the model lake. Again, this type of information is nearly impossible to obtain from the prototype since wind conditions are constantly changing and the water mass is in a continual state of readjustment to these changing external conditions. The conditions for this second study more closely approximate the time-averaged current structure of the lake since the prevailing wind pattern is from the West.

Still unknown, of course, is what the steady-state circulation pattern for the condition of no wind shear but with throughflow would be when the lake is stratified. Similarly, the response of the two-layered lake to a westerly wind is not known. The shallowness of Lake Erie prevents the development of an upper warmer layer (epilimnion) over the entire lake surface and intermittent thermocline structure has been reported for the Western and Central Basins (FWPCA Report, 1968). In addition, the shallowness of the physical model has not encouraged experimentation with stratification. A larger and deeper model would provide a better facility for the study of circulation in the lake during simulated summertime stratification.

VI.2 Theoretical Considerations

Several idealized lake models have been employed by different authors in their studies of the circulation in lakes. These have included a circular basin with constant depth [Csanady (1968)], a circular lake with a parabolic bottom profile [Birchfield (1967)], a rectangular basin of constant depth [Yuen (1968)], and a rectangular lake having a simple variable depth profile [Liggett and Hadjitheodorou (1969)]. For the case of Lake Erie, the basin can be approximated by a rectangular basin of constant depth with a length to width ratio of 4 (see Fig. 10). The geometrical configuration of islands and depth changes cause the solution to the problem to become quite tedious, although attempts are being made to obtain solutions for this more complex situation.

The two basic approaches used in studying circulation in a large basin have been the so-called "inertial theory" and the "frictional theory." In the first approach the friction terms are dropped from the equations of motion and the convective accelerations or inertial terms are retained. In the second, or latter approach, the acceleration or inertial terms are dropped and the frictional term representing the vertical diffusion of momentum is retained. Fofonoff (1954) has utilized the "inertial theory" to study circulation in a homogeneous frictionless sea of rectangular shape. Ekman (1905) first introduced the "frictional theory" which was later generalized by Welander (1957). Csanady (1967) has utilized Welander's linearized equations of motion to study circulation in the Great Lakes.

Calculated Rossby numbers for the Great Lakes are on the order of 10^{-2}. Therefore, the inertial effects are, in general, small enough to be ignored. Presumably, the inertial effects would be significant in

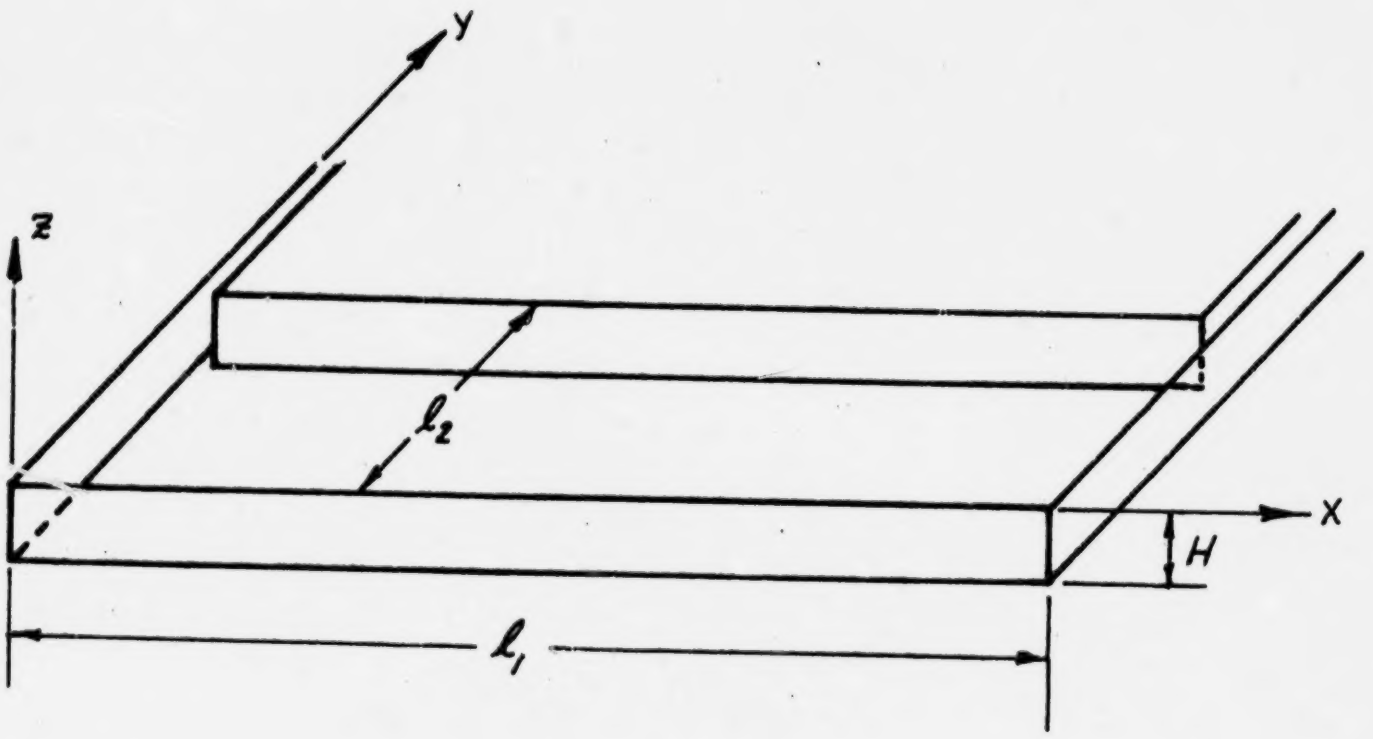


Fig. 10 Idealized Rectangular Lake Basin

the vicinity of inlets where the incoming momentum of the flow is important. The shallowness of Lake Erie indicates that frictional effects may be important and the vertical diffusion of momentum should be included. The horizontal diffusion of momentum is of less importance except where large horizontal variations occur in the velocities such as near lateral boundaries or near inlets. Limited knowledge exists on the vertical distribution of the eddy viscosity, ϵ_z, and to make the mathematics more tractable, most investigators consider ϵ_z to be a constant. Hamblin (1969) has examined these simplifying assumptions and their effect on the steady state circulation in a rotating lake of circular plan including both the throughflow and wind shear.

In this study, the circulation patterns in an idealized rectangular model lake are studied employing both the "frictional theory" and the "inertial theory."

VI.2.1 Frictional Theory

The steady state equations of motion, neglecting the inertial terms and the horizontal diffusion of momentum can be written as

$$-fv = -g\frac{\partial \eta}{\partial x} + \frac{\partial}{\partial z}\left(\epsilon \frac{\partial u}{\partial z}\right) \tag{6-1}$$

$$fu = -g\frac{\partial \eta}{\partial y} + \frac{\partial}{\partial z}\left(\epsilon \frac{\partial v}{\partial z}\right) \tag{6-2}$$

and the continuity equation as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{6-3}$$

In the above equations, u, v, and w are the velocity components in the x, y, and z coordinate directions; f is the Coriolis parameter equal to $2\,\Omega \sin \phi$, η is the water surface displacement from calm, and ϵ is

the vertical eddy viscosity. The hydrostatic assumption requires that the horizontal derivatives of the pressure field be independent of depth. The free surface condition and no-slip bottom condition are:

$$z = 0, \qquad \epsilon \frac{\partial u}{\partial z} = \tau_x, \qquad \epsilon \frac{\partial v}{\partial z} = \tau_y \tag{6-4}$$

$$z = -H, \qquad u = 0, \qquad v = 0$$

In Eq.(6-4), τ_x and τ_y are the components of the surface wind stress. H is the depth of the basin as shown in Fig. 10. The surface profile displacement, $\eta(x,y)$, is assumed small compared to the depth, H.

Introducing the depth-integrated horizontal velocity components,

$$U(x,y) = \int_{-H}^{0} u\,dz, \qquad V(x,y) = \int_{-H}^{0} v\,dz \tag{6-5}$$

it follows that eq.(6-3) becomes

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{6-6}$$

Eq.(6-6) provides for the existence of the transport stream function, ψ, such that

$$U = -\frac{\partial \psi}{\partial y}, \quad V = \frac{\partial \psi}{\partial x} \tag{6-7}$$

For a constant depth basin in which ϵ is constant, the equations of motion reduce to a Poisson equation for an arbitrary wind stress at the free surface, i.e.,

$$\nabla^2 \psi = G(x,y) \tag{6-8}$$

where

$$G(x,y) = \frac{1}{fE}\left[CE + DF\right)\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right) + \left(DE - CF\right)\left(\frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y}\right)]$$

and

$$C = A \sin\frac{\pi H}{D} \sinh\frac{\pi H}{D}$$

$$D = A \cos\frac{\pi H}{D} \cosh\frac{\pi H}{D} - 1$$

$$E = A\left(\sin\frac{\pi H}{D}\cos\frac{\pi H}{D} - \sinh\frac{\pi H}{D}\cosh\frac{\pi H}{D}\right)/\left(\frac{2\pi H}{D}\right)$$

$$F = 1-A\left(\sin\frac{\pi H}{D}\cos\frac{\pi H}{D} + \sinh\frac{\pi H}{D}\cosh\frac{\pi H}{D}\right)/\left(\frac{2\pi H}{D}\right)$$

$$A = 1/\left(\cos\frac{2\pi H}{D} + \cosh\frac{2\pi H}{D}\right)$$

$$D = \pi\sqrt{\frac{2\varepsilon}{f}}$$

Lai and Rumer (1970) obtained solutions for eq.(6-8) for the general case with arbitrary distribution of inlets and outlets on the lateral boundaries of a rectangular basin and for arbitrary wind patterns over the lake surface. The special cases of uniform wind stress, constant curl, and constant divergence were examined in more detail as well as the case including throughflow with and without wind and with complete ice coverage. The solutions indicated the existence of a strong bottom return flow and Ekman spiral for all cases. Although the theory permitted no net flow across the lateral boundaries, it did not require the velocity components at a given position, z, along these boundaries to go to zero. Considering the no-slip condition along the lateral boundaries and including the horizontal diffusion of momentum would be a significant improvement in the theory. In addition, near the inlets and outlets it is expected that the convective accelerations may become important and would have to be included if possible. Bottom topography is important and can cause local convergence of the flow which, in combination with some of the other neglected features, can give rise to the so-called coastal jet [Birchfield (1967)]. A three dimensional finite element approach is presently underway which includes the no-slip condition on all solid boundaries and both horizontal and vertical diffusion of momentum.

VI.2.2 Inertial Theory

The justification for using the inertial theory may be summarized as follows: (1) the size of the basin is so large and so deep that frictional effects may be neglected and (2) the currents are of a magnitude such that convective accelerations can not be ignored. In a study of circulation induced by throughflow in a rectangular basin, Sarma (1968) found a relatively strong current along the south shore of his basin. It is of interest to discuss the use of the inertial theory for the case of the rectagnular basin with throughflow.

Considering the circulation induced by the throughflow in the idealized model lake shown in Fig. 10, the motion is assumed to be horizontal and friction is neglected. The equations of motion consistent with these assumptions are

$$\frac{du}{dt} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (6\text{-}9)$$

$$\frac{dv}{dt} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} \qquad (6\text{-}10)$$

and the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (6\text{-}11)$$

where p is the pressure, ρ is the mass density of the water and the other terms are as defined before.

Eliminating the pressure, p, from eqs. (6-9) and (6-10) and using the continuity equation, yields

$$u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = 0 \qquad (6\text{-}12)$$

Introducing a stream function, ψ, such that

$$u = -\frac{\partial \psi}{\partial y} \quad v = \frac{\partial \psi}{\partial x}, \qquad (6\text{-}13)$$

gives for eq.(6-12)

$$\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} (\nabla^2 \psi) - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} (\nabla^2 \psi) = 0 \qquad (6\text{-}14)$$

Eq.(6-14) can be written more generally as

$$\frac{d}{dt} (\nabla^2 \psi) = 0. \qquad (6\text{-}15)$$

which for steady flow reduces to

$$\nabla^2 \psi = g(\psi) \qquad (6\text{-}16)$$

where $g(\psi)$ is an arbitrary function of ψ.

Eq.(6-16) shows that the vertical component of relative vorticity remains a constant along each streamline. This is an important consequence of the use of the inertial theory in steady two-dimensional inviscid flow.

At present, there is no way to determine the arbitrary function, $g(\psi)$, without analyzing viscous effects. This has been a continuing source of difficulty. Reasonable choices can be made so that particular classes of solutions can be studied with maximum simplicity. A number of linear forms of the function $g(\psi)$ have been chosen in the study of oceanic circulation, such as $g(\psi) = c_o + c_1\psi$ by Fofonoff (1954). The constants c_o and c_1 are functions of the viscous history of the flow and extremely difficult to evaluate. The usual approach is to investigate (in an essentially heuristic fashion) flow fields with given values of c_o and c_1 and to learn what is possible.

Sarma (1968) has computed the stream function for various values of $c_o (c_1 \equiv 0)$ in a rectangular basin with various arrangements for the inlet and outlet. For $c_o = 0.01$, the flow pattern showed a relatively strong current moving eastward along the south shore. See Fig. 11.

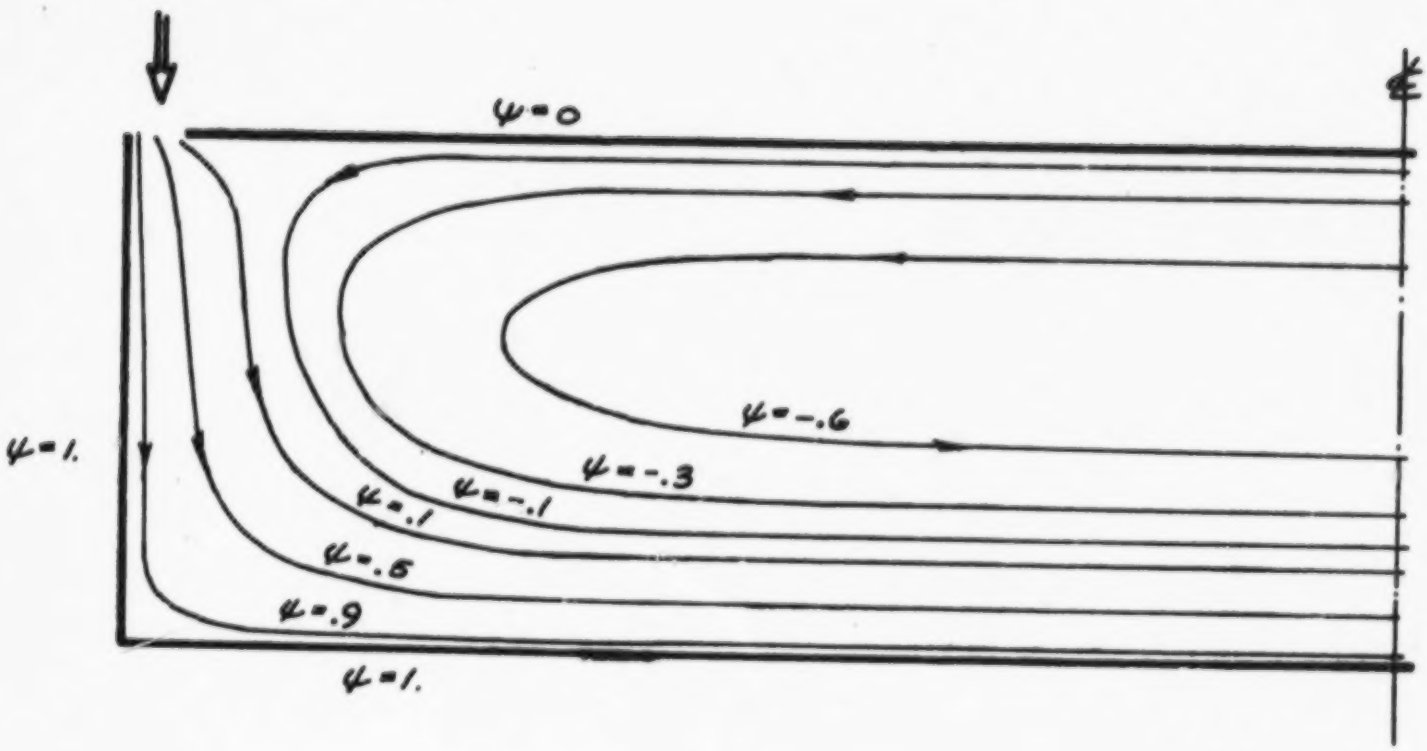


Fig. 11 Flow Pattern in a Rectangular Basin (C_o = 0.01)
(from Sarma, 1968)

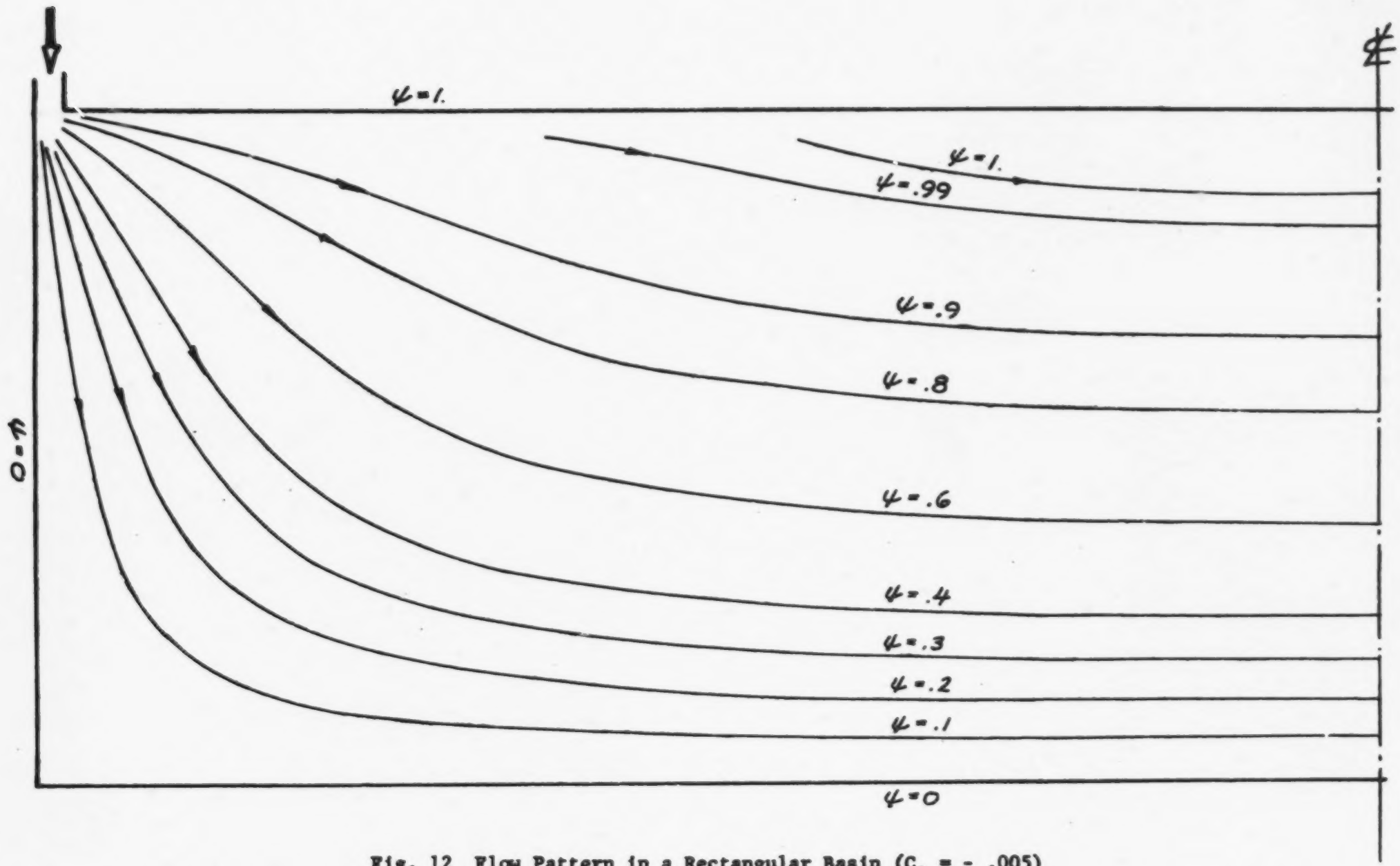


Fig. 12 Flow Pattern in a Rectangular Basin (C_1 = - .005)

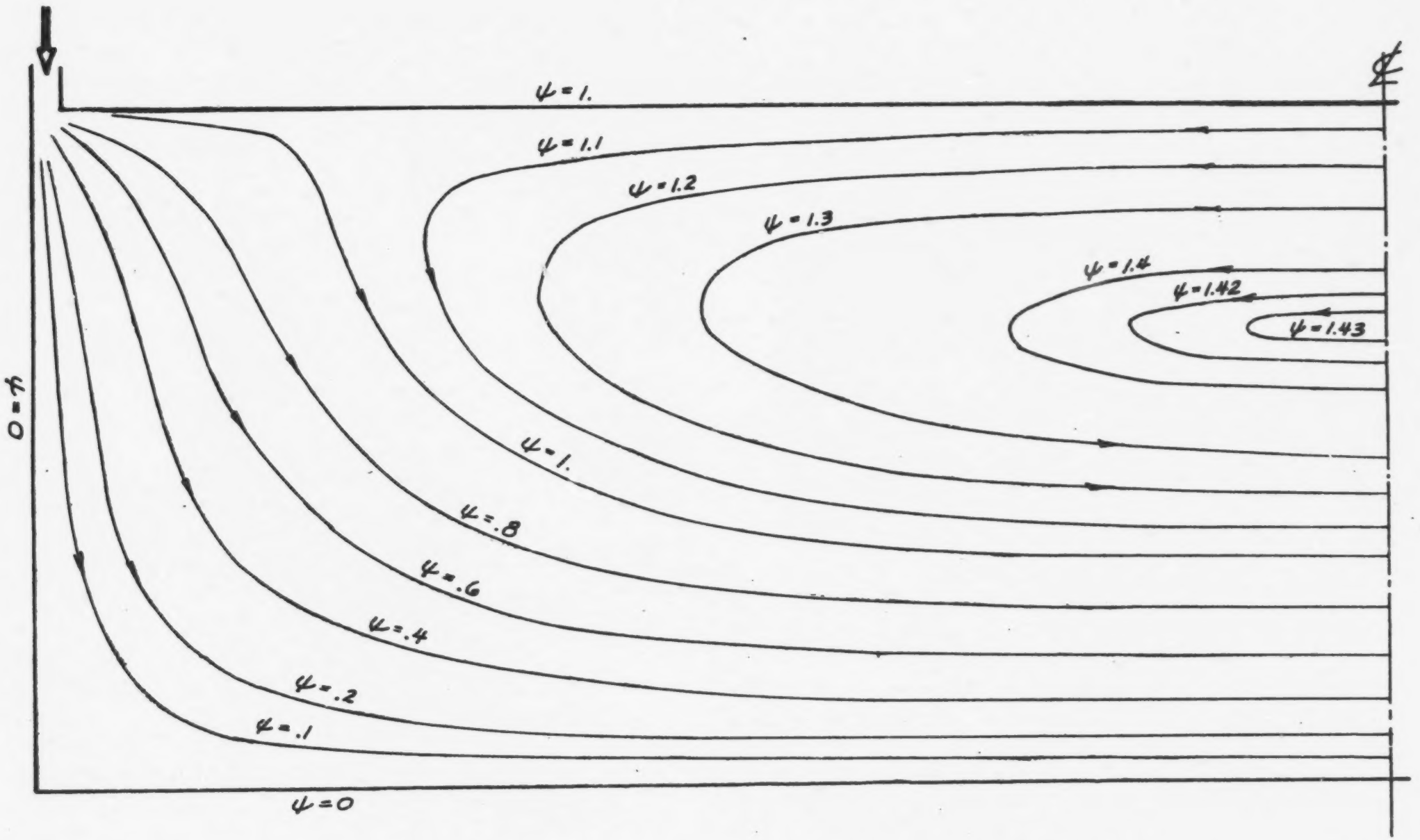


Fig. 13 Flow Pattern in a Rectangular Basin (C_1 = - .01)

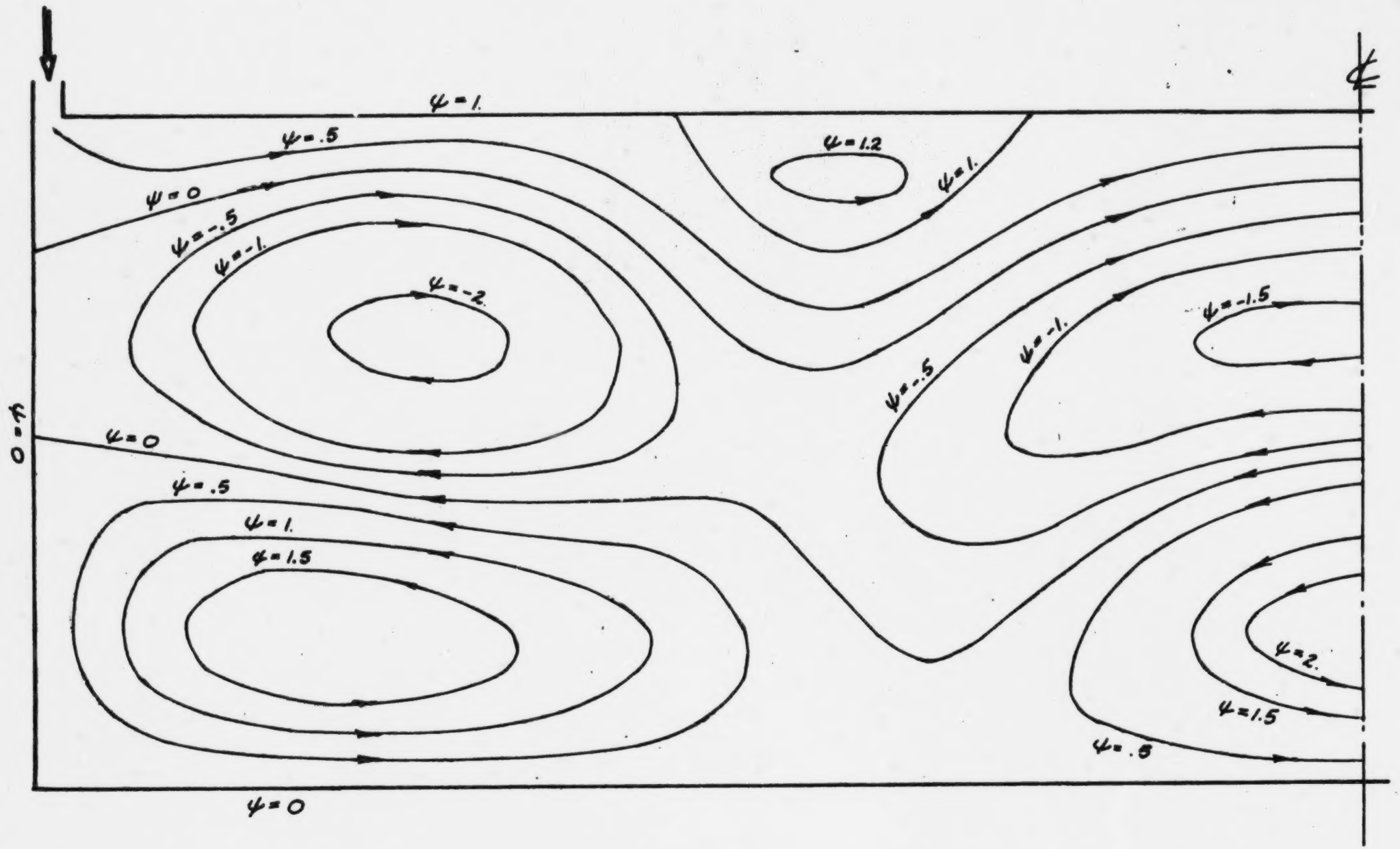


Fig. 14 Flow Pattern in a Rectangular Basin (C_1 = - .1)

If the function $g(\psi)$ is chosen such that

$$g(\psi) = c_1\psi \tag{6-17}$$

then eq.(6-16) becomes

$$\nabla^2\psi = c_1\psi. \tag{6-18}$$

Assuming a uniform velocity distribution at both inlet and outlet, the solution of eq.(6-18) in dimensionless form is given as

$$\psi(x,y) = \frac{1}{24} \sum_{n=1}^{\infty} D_n \sinh\sqrt{\left(\frac{(2n-1)\pi}{96}\right)^2 + c_1}\, y \sin\frac{(2n-1)\pi}{96}\, x \tag{6-19}$$

where

$$D_n = \left(\frac{96}{(2n-1)\pi}\right)^2 \frac{\sin\frac{(2n-1)\pi}{96}}{\sinh\left(24\sqrt{\left(\frac{(2n-1)\pi}{96}\right)^2 + c_1}\right)}$$

and $0 \leq x \leq 96$, $0 \leq y \leq 24$. Eq. (6-19) is plotted in Figs. 12, 13, and 14 for various values of c_1. Again, only the flow in the western half of the basin is given since the pattern is symmetrical about the centerline axis.

The results of these two models using the inertial theory are presented because they have certain similarities to the observed flow in the rectangular basin.

VI.3 Experimental Results: Rectangular Basin

VI.3.1 Without Wind

A rectangular basin of constant depth was constructed from 3/4" thick marine plywood. The bottom of the model was adjusted according to equation (2-20) and was made of 1/4" thick marine plywood. Using a L_r equal to 1:212,000 and a Y_r equal to 1:530, the dimensions of the idealized model lake were 6 ft. by 1.5 ft. The depth of water was 1.5 inches. Also included was an inlet for the Detroit River inflow and an outlet for the Niagara River outflow (see Fig. 10). These two

flows were set equal for the experiments. The rotation rate of the laboratory was 4.4 RPM in accordance with eq. (2-17).

The results of the experiments are depicted graphically in Fig. 15. The Detroit River inflow moves southward with a small counterclockwise eddy that fed back into the main flow. The most predominate feature of the flow pattern is the southshore current. Because of its greater speed, this southshore current represents a short circuiting of at least a portion of the flow through the basin.

The subsurface flow differed from the surface flow in that whereas the surface flow moves rather uniformly toward the outlet, the subsurface flow is characterized by a large counterclockwise eddy. As a result of this, the surface and bottom flows are different in the Northern half of the basin but similar in the southern half. The weak counterclockwise eddy near the inlet was exhibited by both the surface and the subsurface flow.

VI.3.2 With Wind

A study of the wind driven circulation was made in the rectangular basin similar to the studies of Buechi and Rumer (1969). The battery of centrifugal fans were operated at maximum output providing a model wind speed of approximately 15 fps. A qualitative portrayal of the observed circulation pattern is shown in Fig. 16.

With the exception of two eddies that formed near the inlet and outlet, the surface flow was to the right of the wind direction. This surface flow converged along the southern shore and seemed to intensify the southshore current that had been observed in the experiments without wind. The return bottom flow was observed to be weaker than the surface flow velocities. The southshore current was also exhibited by

Surface Pattern

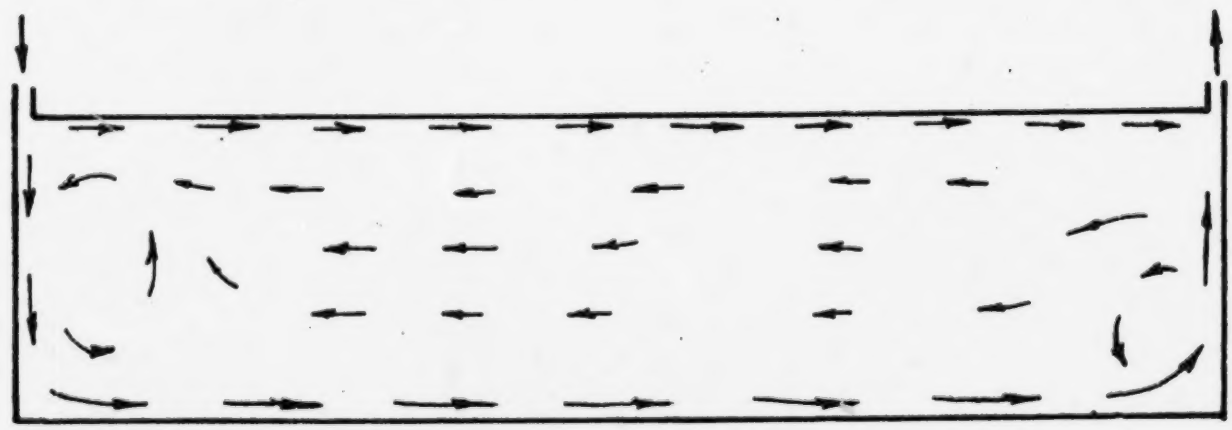
Wind

ω_M = 4.4 rpm

Q = 115 cc/min

V = 15 fps

Vectors indicate direction onl



Subsurface Pattern

Fig. 16 Observed Circulation Pattern in Rectangular Basin (With Wind)

the bottom flow. Areas of upwelling occurred along the northern boundary and sinking occurred along the southern boundary.

VI.4 Experimental Results: Lake Erie Model

VI.4.1 Without Wind

The experimental study of the circulation in the model of Lake Erie was conducted with the inflow of the Detroit River set equal to the outflow of the Niagara River. The model flow was set equal to a prototype discharge of 175,000 cfs.

The results of these studies [Rumer and Robson (1968)] show a relatively well-mixed western basin dominated by a large counter-clockwise eddy generated by the inflow from the Detroit River. The primary flow moves northward and exits the western basin through Pelee Passage. Portions of the flow entering via the Detroit River exit the western basin through South Passage. The flows exiting the western basin move southward collecting along the southern shore and moving eastward along the south shore to the Niagara River (see Fig. 17). Secondary flow patterns are observed in the central basin, the most significant observation being a slow westward drift that fed a stronger eastward return flow along the north shore. The eastern basin was dominated by converging flow towards the Niagara River. Typical water surface velocities measured in the model scaled up to prototype speeds of approximately 0.4 fps. Higher velocities were observed near the Niagara River and in the western basin.

The continuous dye injection studies confirmed the above observed circulation pattern (see Fig. 18). Velocity traverses across the basin from the south shore to the north shore accounted for the bulk prototype flow of 175,000 cfs.

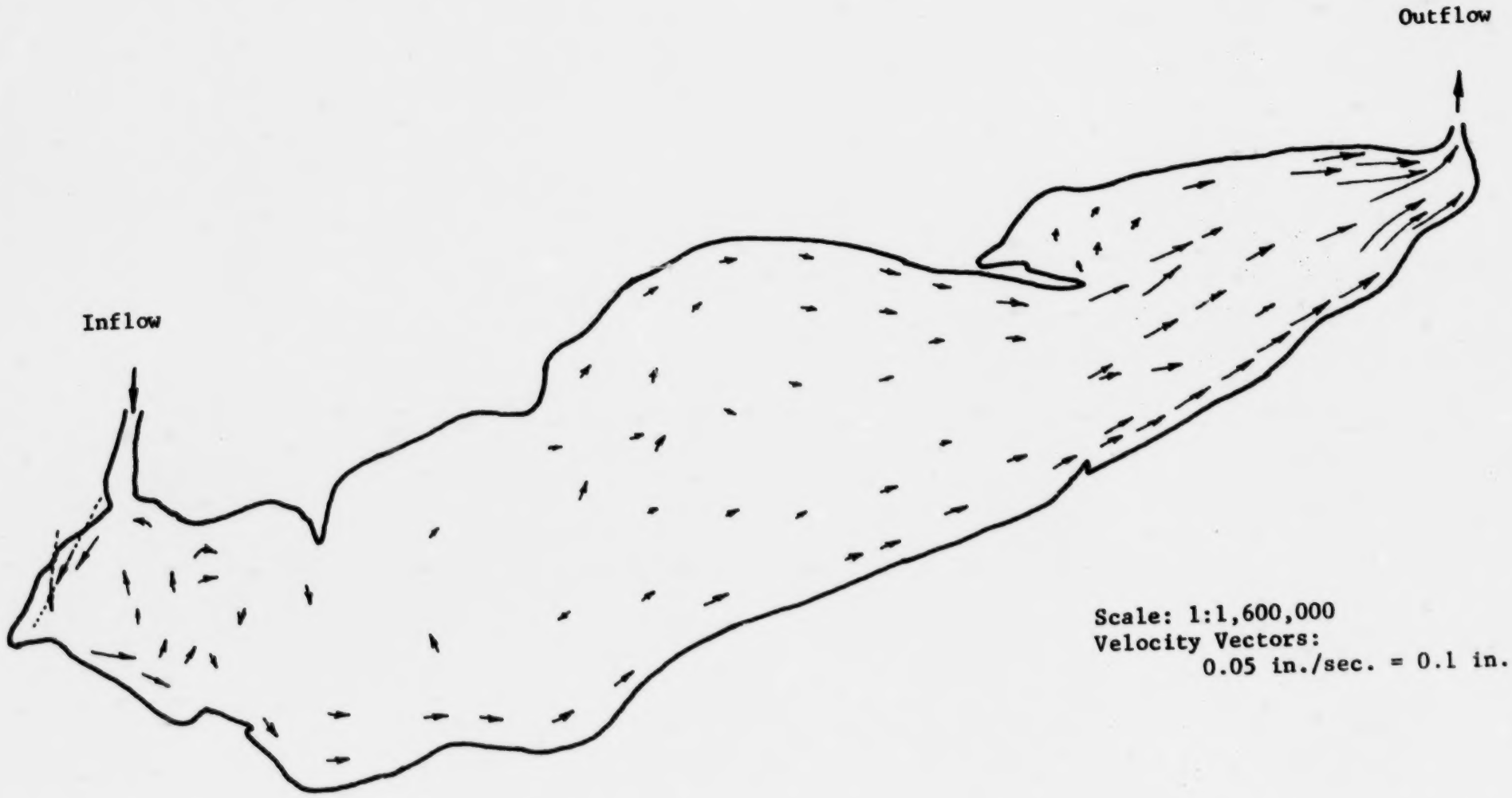
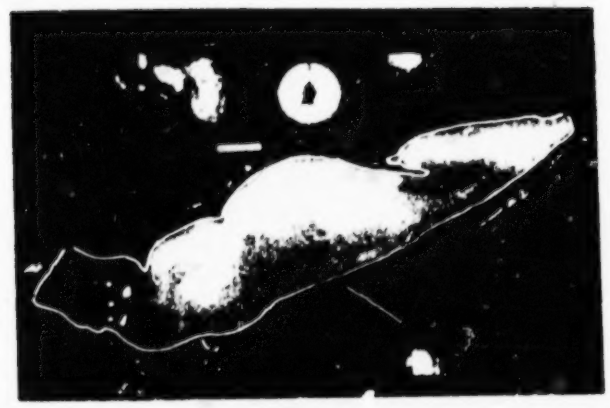
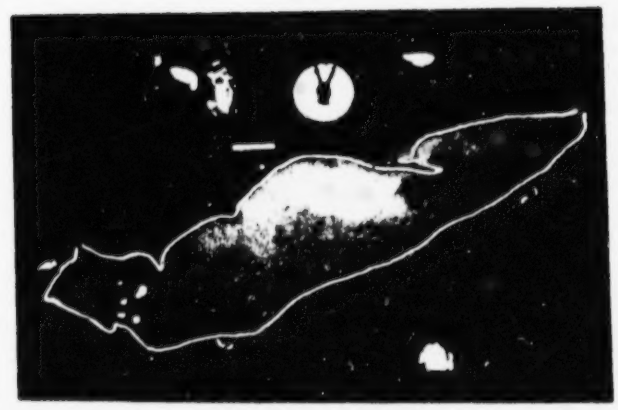


Fig. 17 Observed Circulation Pattern in Lake Erie Model
(Without Wind)



t = 5 min.

t = 19 min.



t = 43 min.

Fig. 18 Dye Path for Continuous Injection through the Detroit River after 5 min., 19 min., and 43 min. (Without Wind)

VI.4.2 With Wind

VI.4.2.1 Wind Set-up

The procedure for determining prototype wind speeds has been described earlier in Part I of this report and elsewhere [Buechi and Rumer, 1969]. A discussion of the results follows. Fig. 19 shows the deviation from calm at Buffalo for different variac settings. These water level deviations were obtained by multiplying the model lake set-up by the vertical scale ratio of 500. Using a plot of the deviation from calm versus wind speed and direction obtained from the Hydro-Electric Power Commission of Ontario, it was possible to assign prototype wind speeds to the above mentioned water level deviations. The result of this procedure is shown in Fig. 20. Since it is assumed that the horizontal motions of a distorted Froude model have been properly scaled, the water surface slopes should also be properly scaled in order to provide similarity between gravitational forces in model and prototype. Thus, the use of water level deviations should be a valid procedure for determining prototype wind speeds.

Eq. (4-3) of Keulegan has been plotted in Fig. 21 along with the experimental data from this study. This plot relates prototype wind set-up, S, to prototype wind speed, V. The velocity used in Keulegan's equation to calculate set-up was modified to account for the difference between a westerly wind and a wind direction along the longitudinal axis of the lake. The agreement is satisfactory and lends support to the prototype data provided by the Hydro-Electric Power Commission of Ontario. Keulegan had earlier compared his equation (4-3) with observed prototype wind set-ups in Lake Erie and found reasonable agreement.

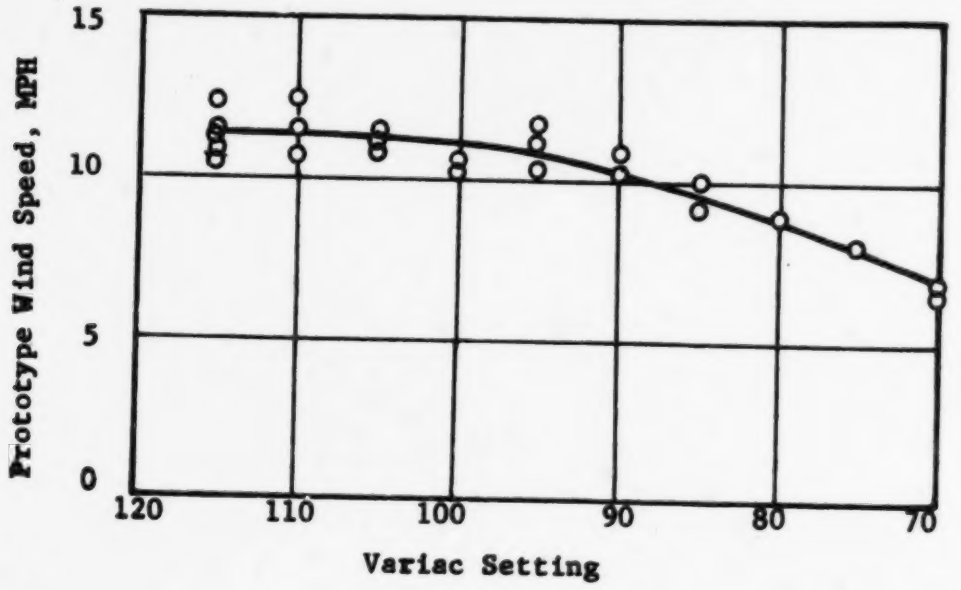


Fig. 20 Prototype Wind Speed versus Variac Setting

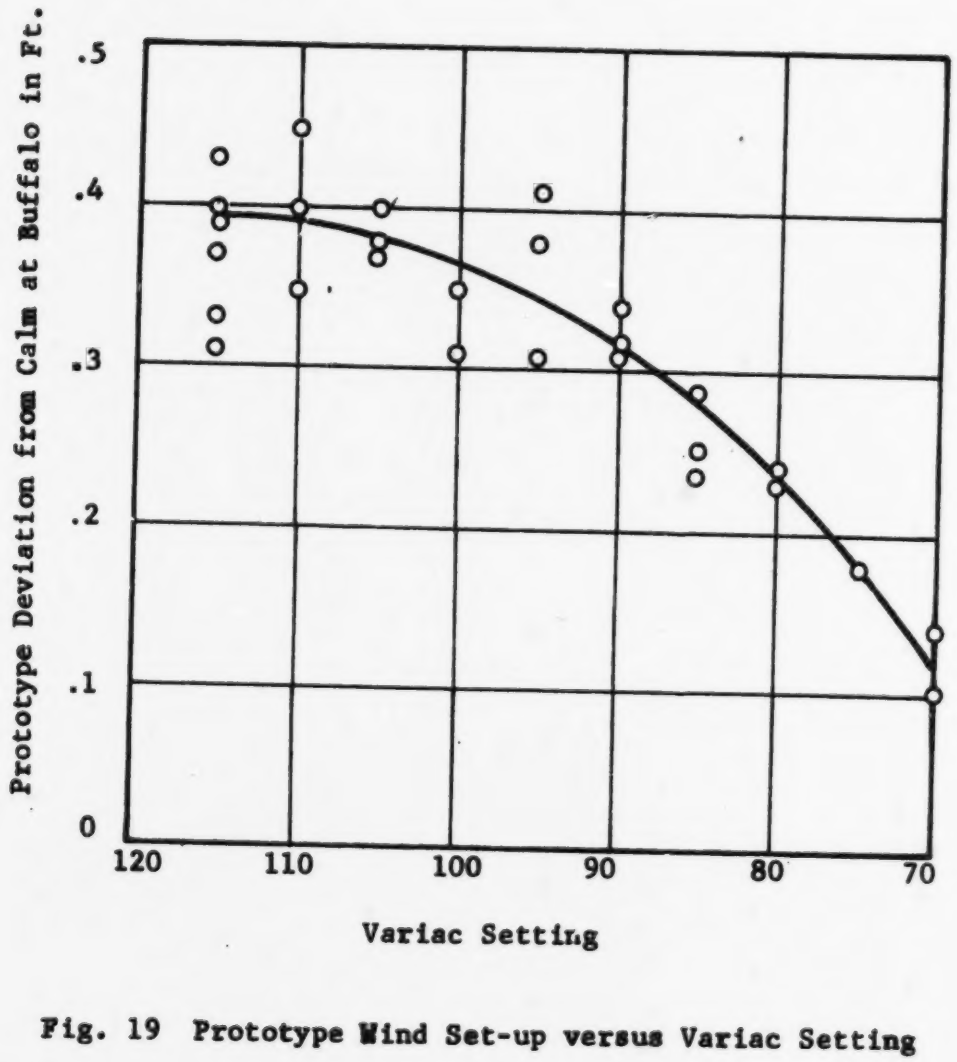


Fig. 19 Prototype Wind Set-up versus Variac Setting

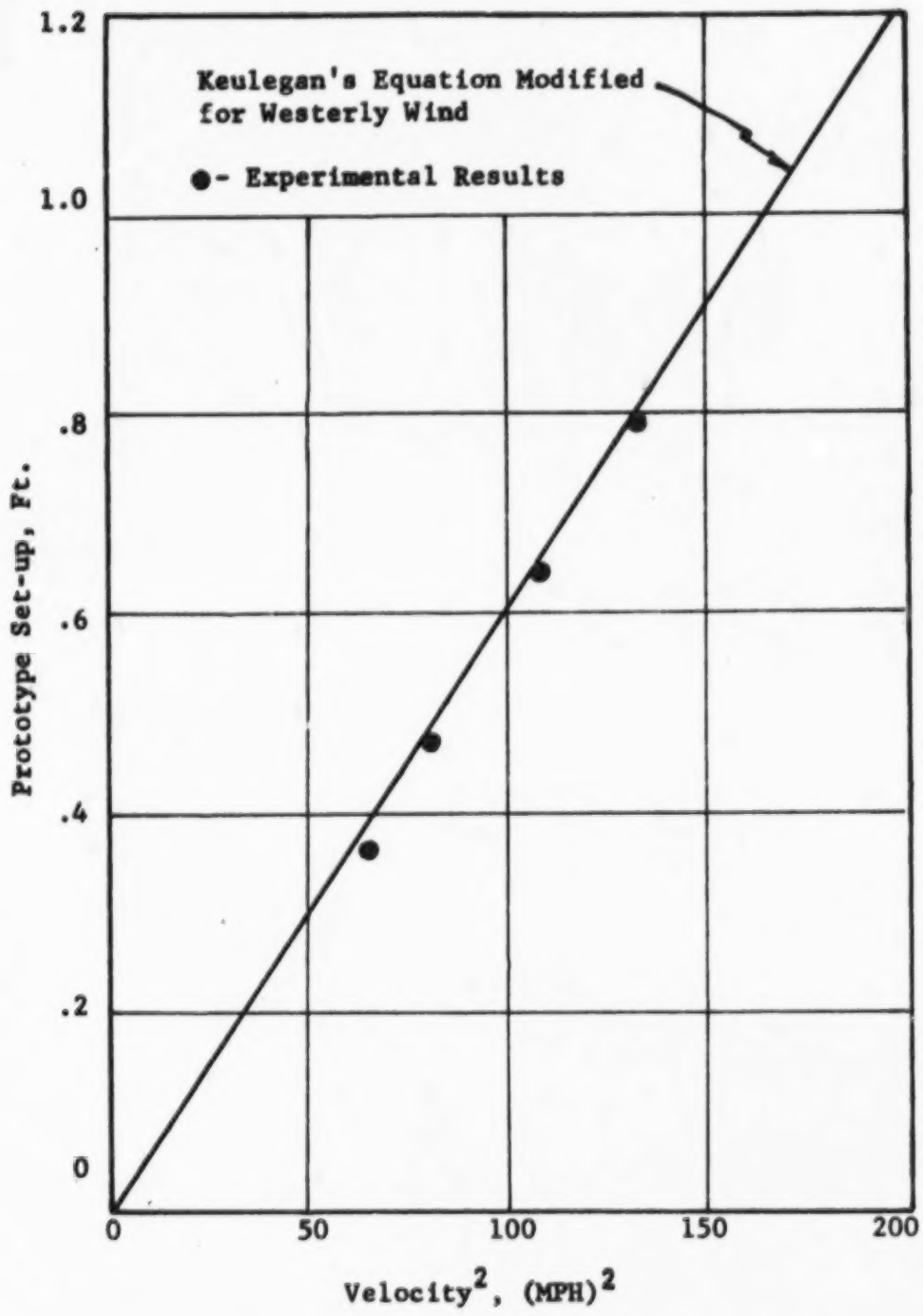
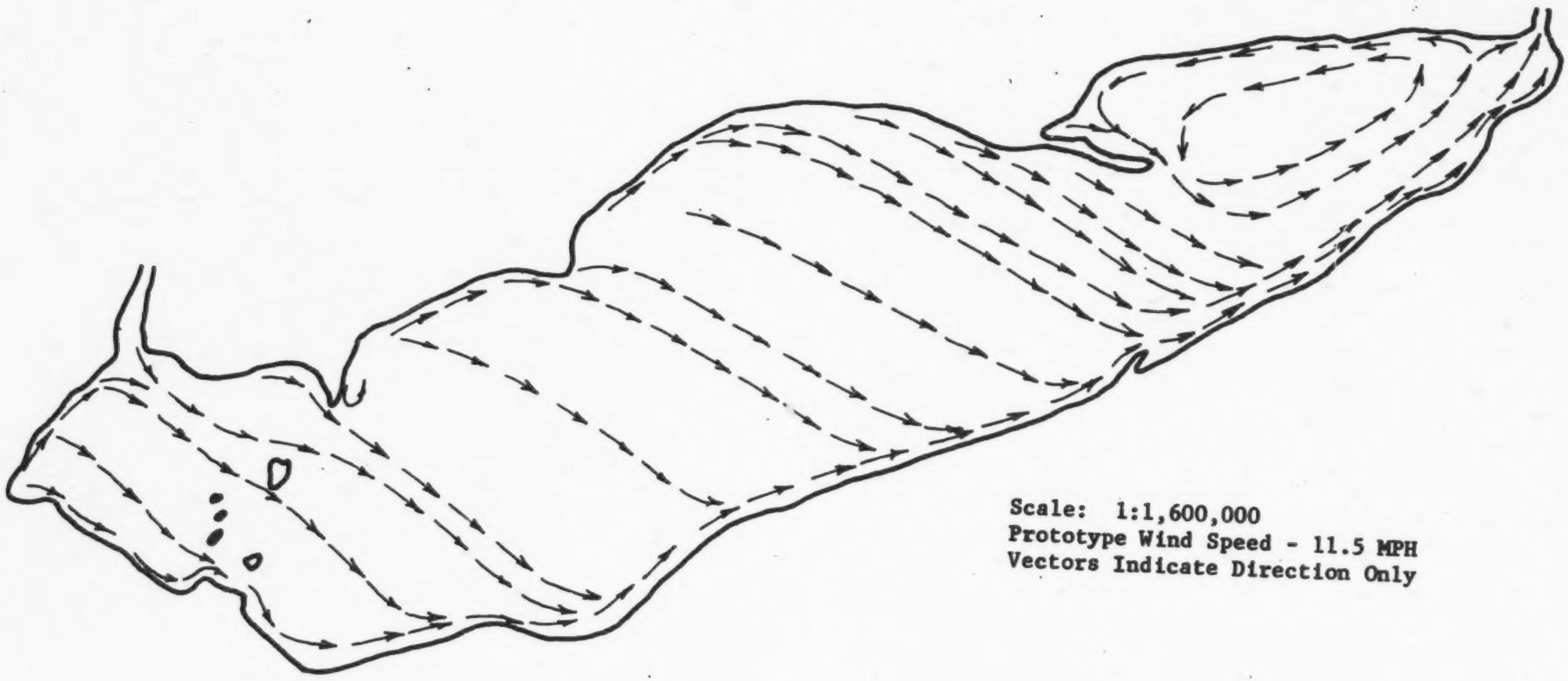


Fig. 21 Relationship Between Prototype Wind Speed and Set-up

VI.4.2.2 Wind Driven Circulation Pattern

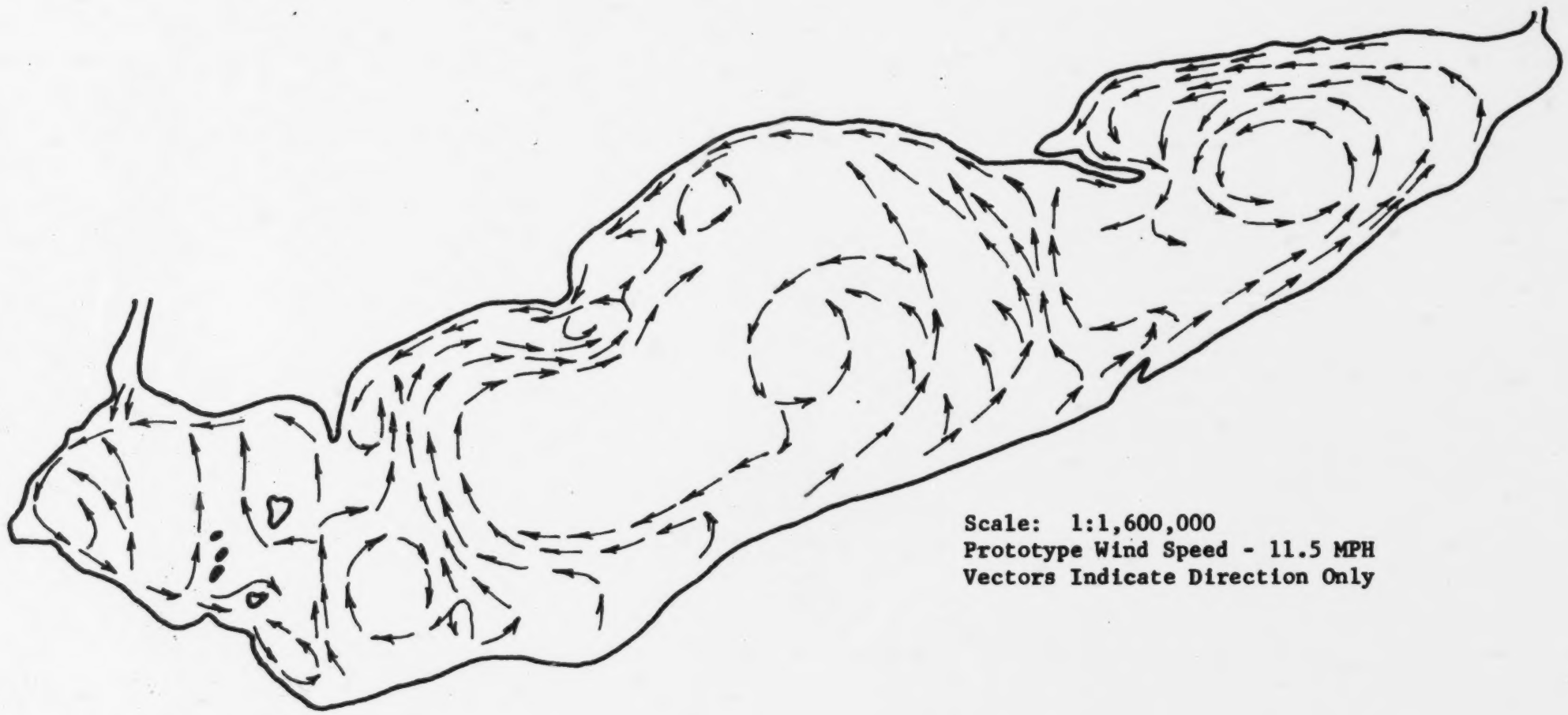
The surface and subsurface circulation patterns existing in the presence of a westerly wind are shown in Figs. 22 and 23 respectively. The vectors represent direction of flow only, not magnitude. The surface pattern is dominated by a southeasterly flow with a strong southshore current in the western and central basins and a large counterclockwise gyre in the eastern basin. The subsurface pattern is more complex, and marked by several stable gyre formations and transverse flows from south to north across the lake.

One significant difference between the wind driven circulation and circulation obtained for zero wind stress that became apparent immediately, was the development of a surface and subsurface pattern for the wind driven case. The presence of surface and subsurface patterns for the wind driven case was not unexpected. Baines and Knapp (1965) noted the existence of jets near the surface and bottom of their channel, moving in opposite directions. An examination of studies concerned with circulation patterns existing in the prototype reveals that both surface and subsurface circulation patterns are predicted. The results reported in the FWPCA (1968) study are shown in Figs. 24 and 25. The vectors represent direction of flow only, not magnitude. It should be noted that both the surface and bottom circulation shown in these diagrams are only given as "dominant summer flow patterns." The physical conditions existing when these patterns are dominant can only be estimated. This should be kept in mind when any comparisons are made between the results of model studies and prototype investigations. Also, the bottom flow pattern is restricted to the flow existing above



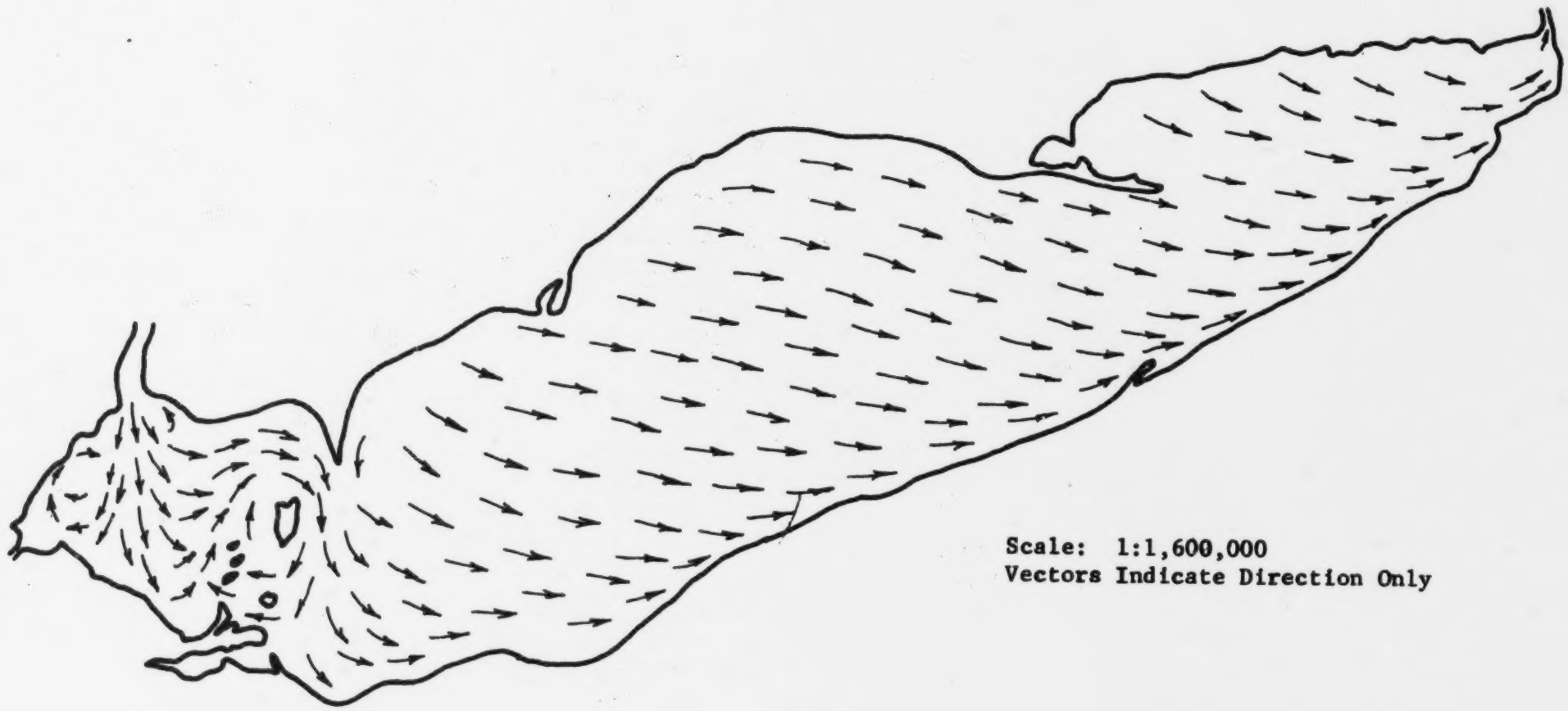
Scale: 1:1,600,000
Prototype Wind Speed - 11.5 MPH
Vectors Indicate Direction Only

Fig. 22 Observed Surface Circulation in Lake Erie Model
(With Wind)



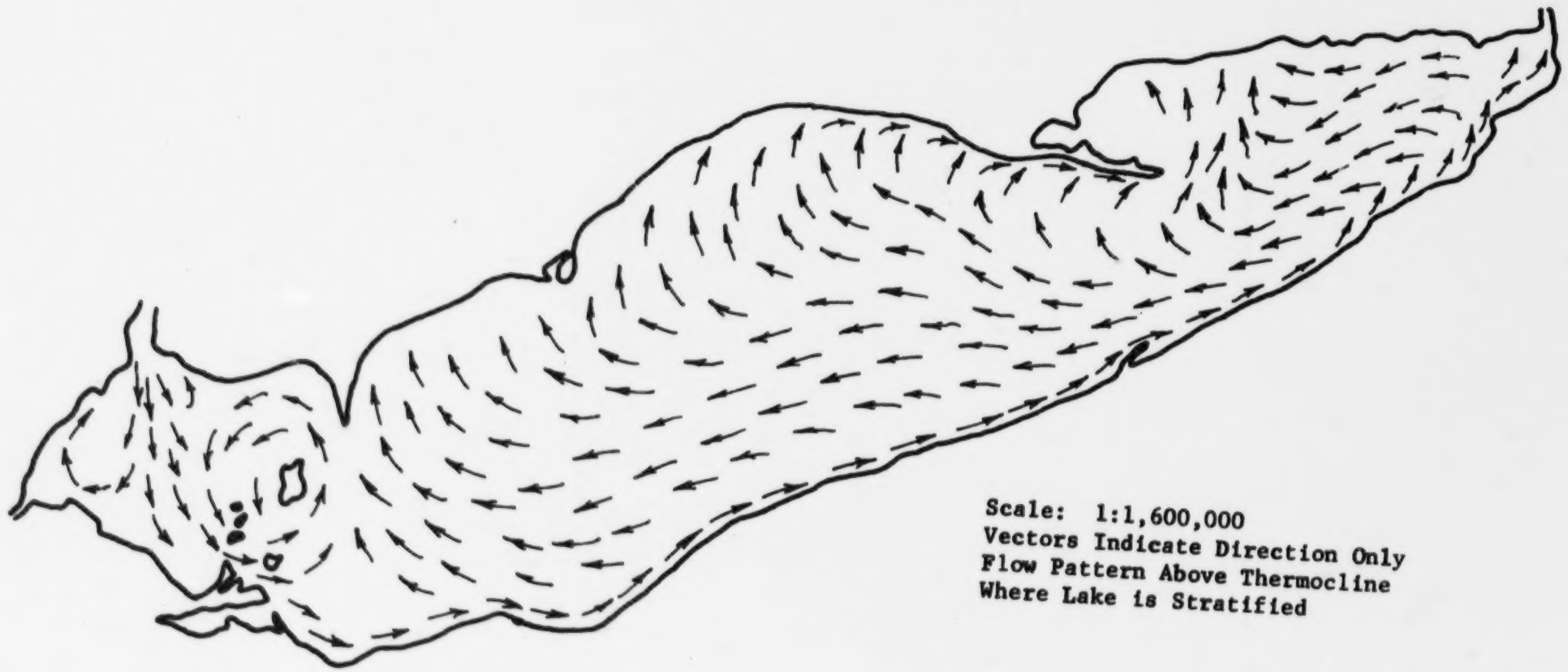
Scale: 1:1,600,000
Prototype Wind Speed - 11.5 MPH
Vectors Indicate Direction Only

Fig. 23 Observed Subsurface Circulation in Lake Erie Model
(With Wind)



Scale: 1:1,600,000
Vectors Indicate Direction Only

Fig. 24 Dominant Summer Surface Flow Pattern in the Prototype
(FWPCA, 1968)



Scale: 1:1,600,000
Vectors Indicate Direction Only
Flow Pattern Above Thermocline
Where Lake is Stratified

Fig. 25 Dominant Summer Bottom Flow Pattern in the Prototype
(FWPCA, 1968)

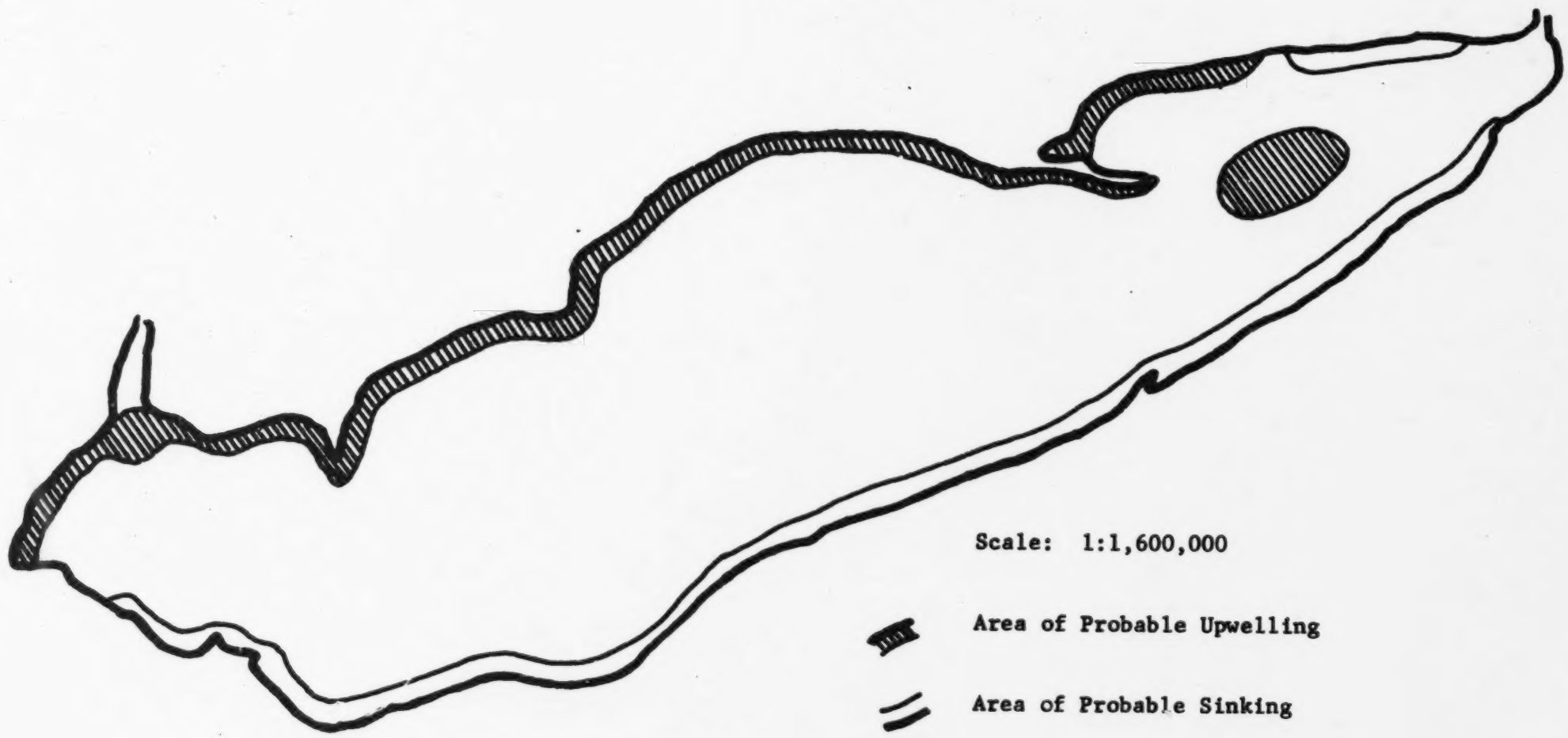


Fig. 26 Observed Areas of Upwelling and Sinking in Lake Erie Model
(With Wind)

the thermocline where the lake is stratified. Since the central and eastern basins of the prototype do become stratified, Fig. 25 can not give a true picture of the bottom flow existing throughout the lake. Possibly the pattern shown in Fig. 25 could be considered as the flow existing in the lake if it was isothermal and had a depth no greater than the thermocline. Even with these qualifications, comparison of the model results with those reported by the FWPCA show many similarities. The surface patterns exhibit only one major difference: a counterclockwise gyre in the eastern basin of the model and not in the prototype. There is less agreement between the two subsurface flow patterns. The greatest difference between the two patterns is the lack of a clockwise current in the eastern portion of the prototype's central basin.

Another significant alteration in the flow pattern is the appearance of areas of vertical motion. Areas of upwelling and sinking in the presence of a westerly wind are shown in Fig. 26. These areas were determined from a study of Figs. 22 and 23 and from test observations.

As noted earlier, the wind driven surface currents in the model were predominantly southeasterly in direction. Since the wind was from the west, the surface currents were deflected to the right. The southshore current observed in the studies without wind still exists for the wind driven pattern, but its velocity has increased significantly.

VI.5 Discussion

The results of this work indicate that the throughflow caused by the inflow of the Detroit River and the outflow of the Niagara River generates significant current patterns in the model basin. Because of the concentration of these currents, observed velocities are often

greater than an average velocity obtained by dividing the total discharge by the cross sectional area. In the absence of wind stress (or probably under ice cover) the Detroit River flow follows a well defined path that carries it first southward and then northward along the western side of the islands. This flow exits the western basin mainly through Pelee Passage but with some flow through South Passage. These flows then regroup along the eastern side of the islands in their flow towards the south shore where the flow then moves along the south shore in an easterly direction towards the Niagara River. In a sense, this south shore current is still representative of the Detroit River inflow under these conditions.

The tendency of the main flow to concentrate along the south shore has been observed in both the rectangular basin and the Lake Erie model. It is reasonable to assume that in the absence of the island chain in the western basin, the Detroit River flow would not turn northward but would flow directly eastward after reaching the south shore. In the management of water quality in the lake, the persistence of this south shore current should be considered as an important factor. The fact that the "inertial theory" can be used to demonstrate the existence of a south shore current is not completely fortuitous. The initial momentum of the incoming flow coupled with the Coriolis effect may very well be the predominant factors in the establishment of this current. Both of these are taken into consideration in the "inertial theory." The tendency for a westward drift in the northern portion of the central basin was observed in the Lake Erie model and in the rectangular model. Fofonoff (1954) observed this in his analytical study of oceanic circulation using the "inertial theory." It is also

predicted by the "inertial theory" analysis in this study. Apparently, this simplified approach can indicate some of the dominant circulation features in the absence of wind shear. A more detailed theoretical treatment of the flow pattern without wind would require the inclusion of the no-slip condition and the diffusion of momentum. The theoretical study of the wind-driven circulation also requires the incorporation of the friction terms in the equations.

The presence of a westerly wind significantly alters the circulation pattern of Lake Erie from the case of no wind stress. The isothermal lake develops somewhat separate surface and subsurface circulation patterns under the action of the westerly wind. These motions are coupled by vertical flows (sinking and upwelling) near the lateral boundaries. The surface pattern is dominated by a southeasterly flow with a strong eastward southshore current. The subsurface pattern is more complex and marked by stable gyre formations and transverse flows north and south across the lake. The procedure used for relating model wind speeds to prototype wind speeds appears promising.

It is apparent that the wind driven surface flow is little affected by the geometry of the basin, actually only by the lateral boundaries. Conversely, the bottom flow under this condition reflects a strong dependence on bottom topography. In the constant depth rectangular basin the bottom flow was more uniform than that observed in the variable depth scale model of Lake Erie. The mass transport from south to north by the bottom flow during the presence of a westerly wind is also worthy of note. Comparison of the observed patterns in the model Lake Erie with the synthesized circulation patterns from prototype observations

[FWPCA(1968)] reveal many similarities and some notable differences. Direct comparison is not possible because the portrayal of prototype circulation patterns is not the measured response of the lake to a westerly wind. Rather, the portrayals are the result of time averaging many observations at several points over many weeks. In that sense, these portrayals are synthesized flow patterns. Since the prevailing wind pattern over Lake Erie is from the West, the synthesized portrayals of circulation should reflect some of the major effects of this wind, which in fact they seem to do. This discussion is a word of caution against too detailed a comparison.

Continued studies of the circulation in Lake Erie should include more detailed studies of the circulation in the model of Lake Erie, the development of more appropriate mathematical models, and continued observations in the prototype with wise choice given to where these observations should be made. Drift card or drogue studies can reveal more information only if the path is traced with time and the wind conditions recorded simultaneously.

VII DISPERSION OF WASTES

VII.1 Introduction

Pollutants can be introduced into receiving bodies of water by direct discharge from a waste water outfall, by entrance through a natural tributary such as a stream or river, from pollution spills within the lake, and in some instances by groundwater seepage. It is useful to know the paths that pollutants will take as they become mixed and diluted by the natural circulation of the receiving water body. The important processes that serve to mix and distribute the pollutant are convection by the main flow patterns and diffusion or dispersion

by temporal or spatial variations in the velocity field.

The dispersion of substances in rivers and estuaries are examples of this phenomena. Usually, both of these cases can be approximated by considering the average flow in the channel to be unidirectional and sometimes time variant, as in the case of estuaries where tidal flow is important [Holley and Harleman (1965)]. Normally, this approximation can not be made in the case of discharge into lakes and into oceans. The flow patterns in these latter two types of receiving water bodies are influenced not only by gravitational forces, but also by the effect of the earth's rotation and by wind stresses acting at the air-water interface. In addition, variations in the distribution of mass density due to thermal stratification and vertical motion near lateral boundaries further complicate the circulation patterns. As a result, the task of predicting the paths of pollutants, their mixing,and dilution, becomes more difficult.

In Chapter II.2 of Part I of this report the criteria for similarity between model and prototype were examined to determine the feasibility of simulating diffusion and dispersion processes in the model. The results of this examination were somewhat discouraging for the scale model constructed in this investigation. Chabert-D'Hieres and Hyacinthe (1967) constructed a rotating model for studying the dispersion of radioactive wastes occurring in the sea over extensive areas and long periods of time. In their case the model scale ratios were L_r = 1:50,000 and Y_r = 1:500, giving a distortion ratio of 100. Quantitative evaluation was made of the horizontal dispersion coefficient in the prototype based on field measurements. The horizontal dispersion coefficient was scaled according to eq.(2-7) which, for the chosen scale

ratios, gives $(E_{xy})_r = 1:1.12 \times 10^6$. According to eq.(2-8) the ratio of the vertical mass transport coefficient in model and prototype would be given by $(E_z)_r = 1:112$. No attempt was made to simulate vertical mixing by Chabert-D'Hieres and Hyacinthe.

In spite of the limitations regarding similitude of the mixing processes, the horizontal currents are scaled according to the Froude criterion and therefore the convective transport of mass in the model, should be in agreement with prototype behavior under similar conditions. This fact led to the use of the model as a means of studying average detention times for tracers introduced into the lake.

The period of residence of contaminants that are introduced into the lake is of considerable interest and importance to water quality management. The usual approach to determining residence times is the impulse injection of a tracer followed in time by the measurement of the tracer concentration at some point of interest. The downstream measurement provides a history of the spread of the tracer and is often referred to as a breakthrough curve or recovery curve. In unidirectional flows this technique can be an effective way of determining the mixing characteristics of the flow system. As has already been pointed out, it is unlikely that accurate information can be learned regarding the prototype mixing characteristics in the lake model but attention to the peaks of the recovery curves should provide useful information on periods of residence.

VII.2 Detention Period Studies

The series of experiments reported on here are for the condition of zero wind stress with a constant throughflow. The laboratory was

rotated at 4.2 RPM in accordance with the requirement of equal Froude number and Rossby number in model and prototype and sufficient time was allowed for the basin flow to stabilize. Several runs were made at each of the points 1 through 13 (see Fig. 27). For points 1 through 8 and including point 13, rhodamine B dye was used as the tracer and all response curves were measured by passing the Niagara River flow through a fluorometer. For points 9 through 12, dilute aqueous solutions of ammonia (approximately 2%) were introduced into the Detroit River. The response curves for this latter set of runs were measured and recorded by placing a small conductivity probe into the flow at the proper position. The change in conductance of the solution was sensed by the probe and recorded automatically. (See Fig. 28).

The results obtained from these studies are summarized in Table I. It must be kept in mind that these data pertain only to those conditions when wind stress can be assumed unimportant. Because the circulation pattern in the western basin is strongly influenced by the Detroit River inflow, it would be expected that on calm days, the general circulation pattern would approach that observed in the model more quickly than flows in the central basin.

The data presented in Table I have been graphically portrayed as depicted in Fig. 29. The mean residence times (or time to peak) measured at the various positions in the model lake tend to support this portrayal of the path taken by tracers injected into the Detroit River.

Apparently there are three distinct paths by which the tracer arrives at the south shore of the western part of the central basin. These are indicated in Fig. 29. The bulk of the flow exits the

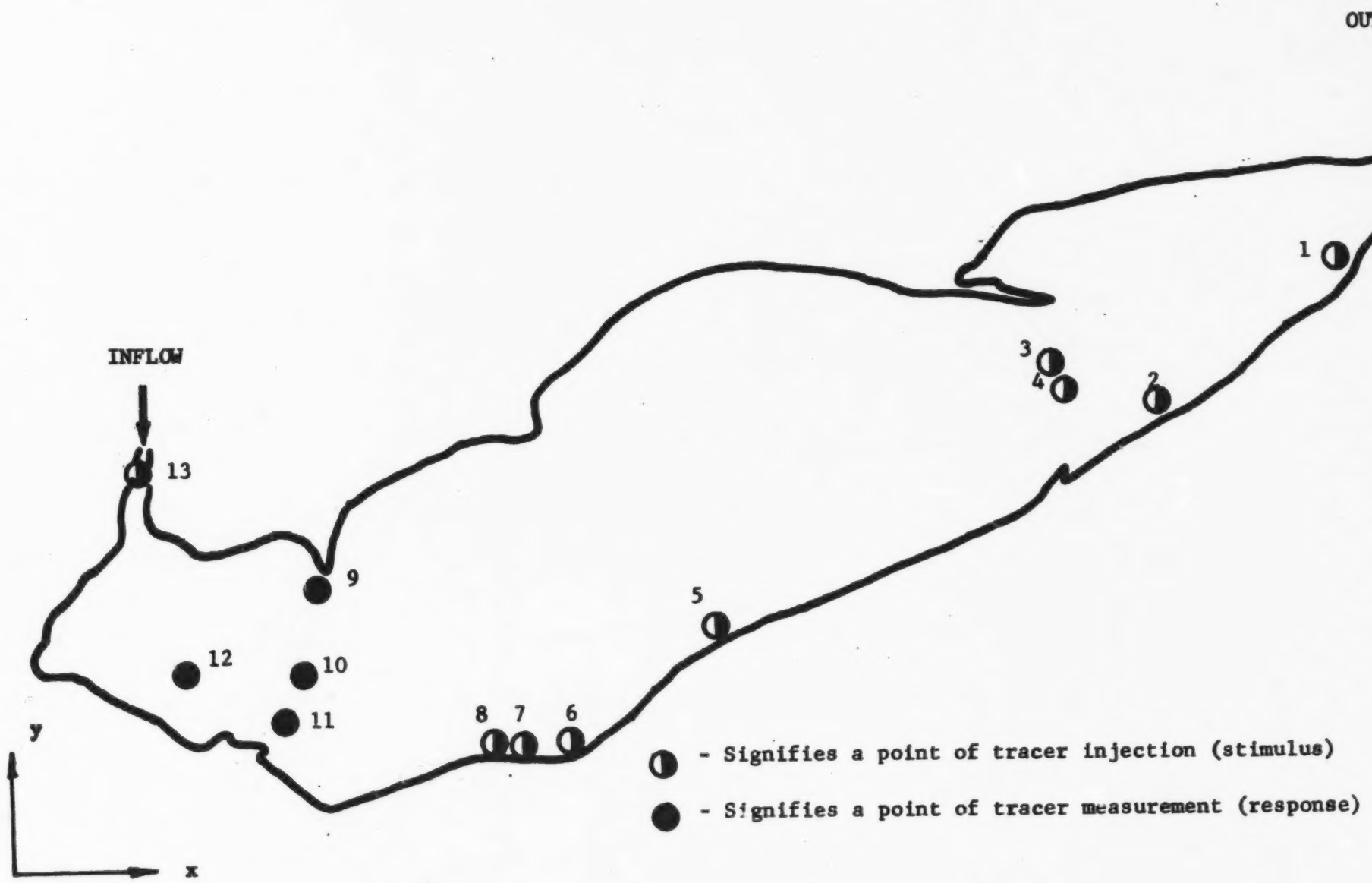


Fig. 27 Location of Points of Injection and Measurement for Tracer Studies in Model Lake (Without Wind)

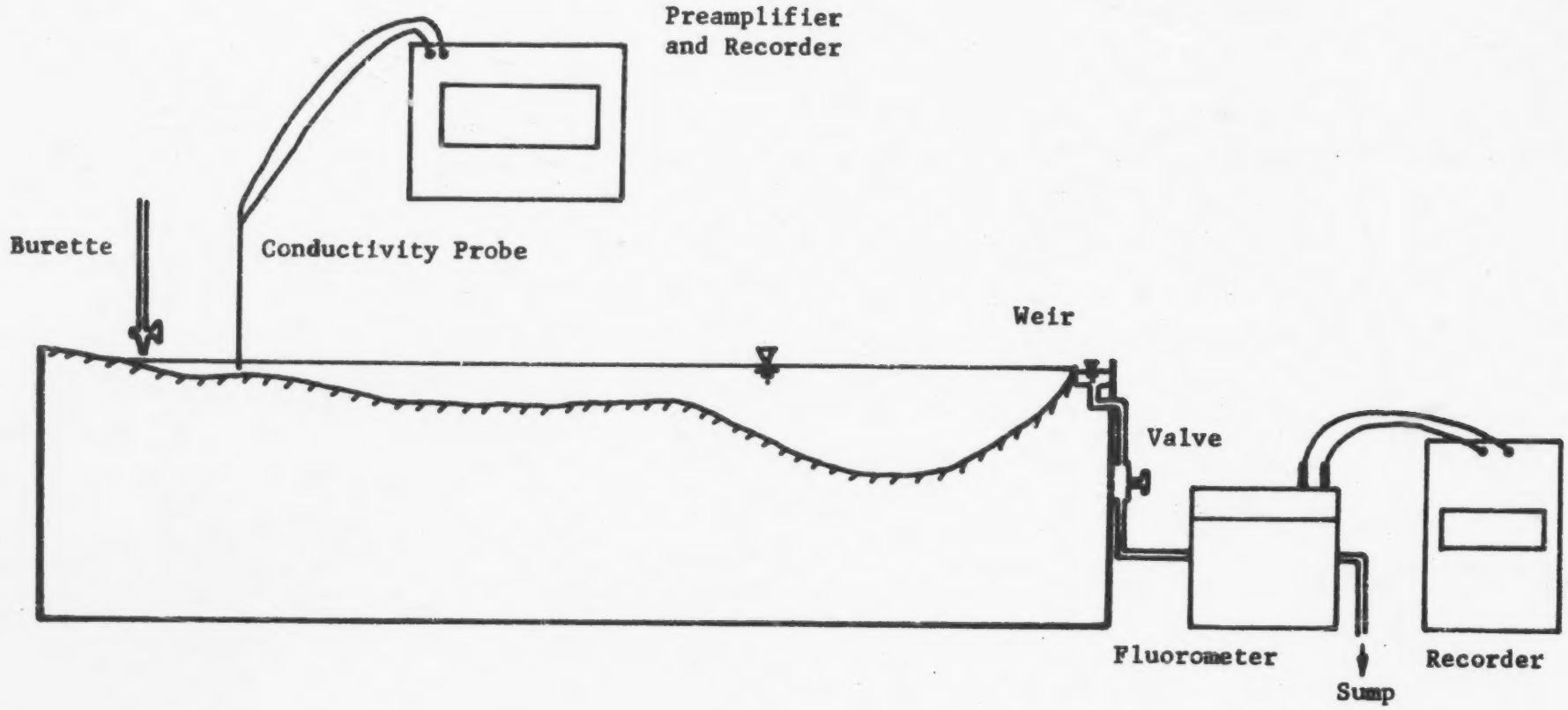


Fig. 28 Experimental Set-up for Tracer Studies

TABLE I

TABULATION OF DATA FROM TRACER STUDIES

POINT	COORDINATES (x,y)*	TRACER	TECHNIQUE OF MEASUREMENT	AVE. TIME TO PEAK IN MODEL (PROTOTYPE)	COMMENTS
1 (injection)	71.6", 29.2"	Rhodamine B	Fluorometer	12 min. (74.4 days)	Response measured at Niagara River
2 "	58.6", 20.7"	"	"	27.1 min. (168 days)	"
3 "	55.08",26.4"	"	"	26.02 min.(161 days)	"
4 "	54.84",25.2"	"	"	37.75 min.(234 days)	"
5 "	36.84",11.28"	"	"	49.9 min. (309 days)	"
6 "	29.82", 5.64"	"	"	58.7 min. (364 days)	"
7 "	26.76", 5.64"	"	"	49.5 min. (307 days)	"
8 "	25.2", 6.0"	"	"	57.4 min. (355 days)	"
13 "	Detroit River	"	"	70.0 min. (434 days)	"
9 (measurement)	16.5", 14.25"	2% soln.NH_3	Conductivity	7.2 min. (44.5 days)	Injection at Detroit River
10 "	15.6", 9.6"	"	"	10.8 min. (67 days)	"
11 "	14.6", 7.25"	"	"	4.4 min. (27.2days)	"
12 "	9.5", 10"	"	"	2.47 min.(15.3 days)	"

*Coordinate distances refer to model lake coordinate system

Observed Flow Pattern in Western Basin

Flow Distribution in Western Basin

Fig. 29 Circulation in Western Basin of Lake Erie Model (Without Wind)

western basin through Pelee Passage. The travel time for the flow exiting through the South Passage is approximately 27 days when scaled up to prototype time. The residence time for the flow exiting through Pelee Passage is approximately 45 days. These flows move southward to join and form the southshore current that persists all the way to the Niagara River. The time of travel for this collected flow as it moves through the central and eastern basins to the Niagara River is approximately 390 days. Thus, the total mean residence time for the entire basin is approximately 435 days. Approximately 10% to 15% of this period can be attributed to the holding time of the western basin. The remainder is the travel time along the south shore to the Niagara River. The average speed of this south shore current as determined from the tracer studies is 0.44 miles per day. This velocity did not vary widely at any point along the south shore.

Often, the holding time of a basin is crudely approximated by dividing the total basin volume by the throughflow. The result of using this approach gives a holding time for Lake Erie of about 1000 days. Such an approach, although providing an order of magnitude, can give misleading values for detention periods.

VII.3 Theoretical Considerations

The equation for the conservation of a dispersing substance is given by the so-called convective-dispersion equation. This equation can be written as:

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z} = \frac{\partial}{\partial x}\left(E_x\frac{\partial s}{\partial x}\right) + \frac{\partial}{\partial y}\left(E_y\frac{\partial s}{\partial y}\right) + \frac{\partial}{\partial z}\left(E_z\frac{\partial s}{\partial z}\right) \qquad (7\text{-}1)$$

where the notation is as defined for equation (2-4). General solutions to this equation are only possible if the velocity field (u,v, and w)

is known and if the dispersion coefficients (E_x, E_y, and E_z) are known or their dependence on the velocity field can be prescribed. In practice, the magnitude of the coefficients will depend upon the character of the flow including temporal and spatial variations.

Solutions to equation (7-1) have been obtained for estuaries where the flow can be reasonably assumed to be one dimensional and the mixing two dimensional. For those cases, the transient effect of tidal mixing is time averaged out of the convective terms and incorporated in the mixing coefficients, E_x and E_z. Until more knowledge is obtained concerning the general velocity field, the use of physical models coupled with semi-empirical mathematical models will be important components to the study of tracer distributions and detention periods.

It is possible to achieve a measure of predictability within certain subregions of the basin where the flow can be approximated mathematically and the mass transport coefficients can be reasonably approximated. This would include regions near river discharges or over small local areas where the flow is essentially unidirectional and the mass transport coefficients have been measured or can be reliably assumed.

The more general problem is rather difficult because of the highly two dimensional nature of the flow pattern, the lack of knowledge concerning mass transport coefficients, and the time dependency of the flow pattern on wind conditions and thermal stratification.

It appeared desirable to develop an empirical description of the results of the physical model experiments. A method for a compact description of the model's circulation patterns has been proposed [Howell, Kiser and Rumer (1970)]. The work of Dorrestein (1960) and Preddy and Webber (1963) was adapted in the development of this

empirical approach. The technique regards time and space as being discrete rather than continuous variables, and considers the movement of matter from one region in space to another over a fixed time interval. The technique was developed only for the western basin under conditions of zero wind stress but with the steady throughflow.

The details of this approach will not be repeated here. The technique utilizes a transition probability matrix to represent in a compact and useable form the flows measured in the model. The technique lends itself to the analysis of nonconservative substances as well as conservative substances. The flow patterns and directions were delineated by injecting a neutrally buoyant dye at various points and along various lines in the western basin of the model. Time lapse motion picture photography was used to obtain the position and motion of the dye over a period of time. The technique may be used to predict the response curve at various points in the basin resulting from injections of a tracer substance at some other point. In this case the transition probability matrix method was used to generate response curves for a tracer substance injected at the mouth of the Detroit River and measured at the collecting point of the different flows exiting from the western basin. The analytical results were in good agreement with experimental results from the model (see Fig. 30). Three peaks appear in the response curves for both the experiments and the analysis and they are virtually coincident. The first peak represents the flow which travels south of Bass Island, turns north of Kelly's Island and enters the region of interest. The second peak is due to the flow which travels northward along the western shores of the Bass Islands, turns sharply southward again and enters the region of interest by passing north of

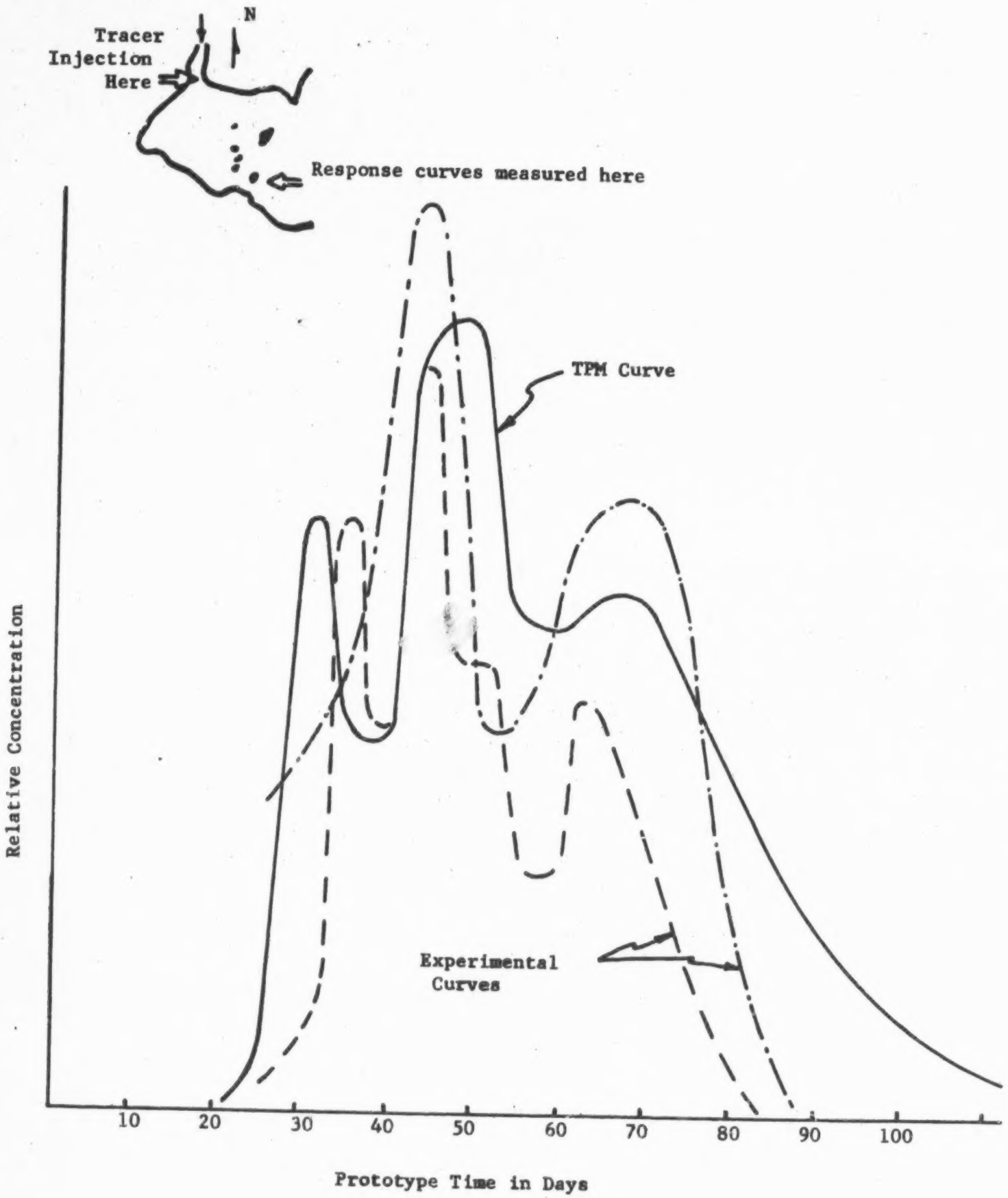


Fig. 30 Response Curves for Tracer Injection in Western Basin
(Without Wind)

Kelly's Island. The third peak is due to the northward flow from the region of the Bass Islands which combines with the flow (fortuitiously after about the same elapsed time) which travels directly along the north shore of the lake from the Detroit River and turns southward east of Pelee Island to enter the region of interest.

VII.4 Discussion

It will require combined use of mathematical modeling and physical hydraulic modeling in conjunction with prototype observations to improve our capability to predict the distribution of contaminants in large lakes. As mentioned earlier, the most serious drawback to solution of the convective-dispersion equation is inadequate information on the structure of the velocity field and insufficient knowledge on the values of the mass transport coefficients. Ideally, this approach should relate the magnitude of the mass transport coefficients to the velocity field in some appropriate manner. It is still useful to pursue this theoretical approach making whatever simplifying assumptions that seem reasonable and that will lead to further understanding of pollutant mixing.

The use of the transition probability matrix technique for describing the distribution of a contaminant caused by lake circulation offers some advantages at the present time. It is compact and it can store information on the large-scale nature of the flow patterns while retaining enough detail to distinguish between different areas of the lake. The major disadvantage is that a different transition probability matrix is needed for each distinct flow pattern that occurs in the lake and therefore it would be necessary to construct as many matrices as

there are flow patterns. Continued development and refinement of this technique could provide the capability for predicting the fate of continuous sources of pollutants discharged into the lake and for randomly occurring sources such as those resulting from "pollution accidents." An aspect that will need further study in relation to this approach is the time necessary for the lake to adjust to a change in external conditions. Thus far, the transition probability matrix is only applicable to steady-state circulation patterns.

Preliminary studies in the physical model for a tracer entering the Detroit River in the presence of a westerly wind of moderate speed (approximately 12 mph.) indicate that the detention period of the basin can be significantly reduced (to less than 30 days). It becomes apparent that consideration must be given to other factors (e.g., the transient effects of varying wind conditions, stratification, ice cover, and seiching) in the final determination of detention periods. It should be noted that there will be points within the lake that will exhibit detention periods longer than the ideal due to the nature of the general circulation patterns.

VIII MASS OSCILLATION

VIII.1 Introduction

The static equilibrium position of a lake surface may be disturbed by sudden changes of atmospheric pressure, wind or local rain storms. When the disturbing force subsides, free oscillations of the water mass ensue. Such oscillations were termed seiches by de Duillier in 1730 in his studies of Lake Geneva (Hutchinson, 1957).

The actual behavior of a lake during seiching is difficult to describe or predict. The difficulty is due not only to the irregular shapes of most lake basins, but also to the various currents and tidal motions that co-exist with the seiches. Moreover, the oscillations are composed of many modes, each of which is modified to a different extent by the frictional forces as well as the earth's rotation. Despite these complications, a general understanding of seiches has emerged from a vast amount of field data and some simplified hydrodynamic theories. It is known that the periods of the free oscillations depend on the configuration of the lake basin, that they increase slightly while the amplitudes decrease under the influence of frictional forces, and that the earth's rotation has only a minimal effect on seiches in most lakes. It is also known that seiches in a lake usually occur along the longest or longitudinal axis of the basin but occasionally transverse. During the motion, every particle of the lake oscillates synchronously with every other, the periods and phases being the same for all.

A general account of the hydrodynamic theories of seiches as well as a bibliography of the early works were given by Chrystal in 1905. More recently, Proudman (1953), Hutchinson (1957), and Defant (1961) have each dealt with this topic.

In this investigation, the seiches in the Lake Erie model were examined as a means of model calibration. Mass oscillations, with an emphasis on their decay, were investigated using the rectangular basin of constant depth with the objective of establishing a technique for scaling friction in hydraulic lake models. Finally, the problem of pollutant mixing during seiching was investigated in a preliminary way.

When the lake undergoes oscillatory motions, the pollutants in the lake must be convected and dispersed at a rate commensurate with the oscillations. How to incorporate this effect in the dispersion equation forms the last section of this chapter.

VIII.2 Free Undamped Oscillations

VIII.2.1 Theoretical Considerations

For a rectangular basin of constant depth, the equations of motion and continuity (assuming that frictional and Coriolis effects can be neglected) can be written as (Hutchinson, 1957)

$$H \frac{\partial u}{\partial x} + \frac{\partial \eta}{\partial t} = 0 \tag{8-1}$$

$$\frac{\partial u}{\partial t} = - g \frac{\partial \eta}{\partial x} \tag{8-2}$$

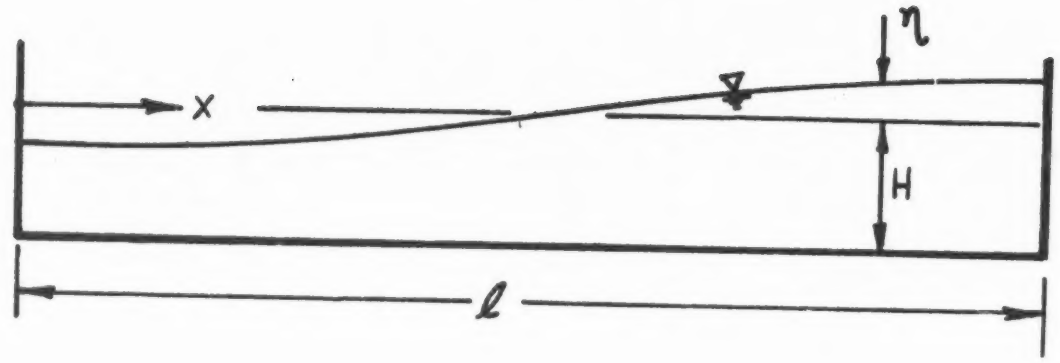
where H is the mean depth of water, u is the horizontal velocity, η is the displacement of the free surface from equilibrium, t is time, and x is the longitudinal direction of the basin (see Fig. 31). Solving these equations and applying the proper boundary and initial conditions results in

$$u = U_m \sin \frac{n\pi x}{\ell} \sin \frac{2\pi t}{T_n} \tag{8-3}$$

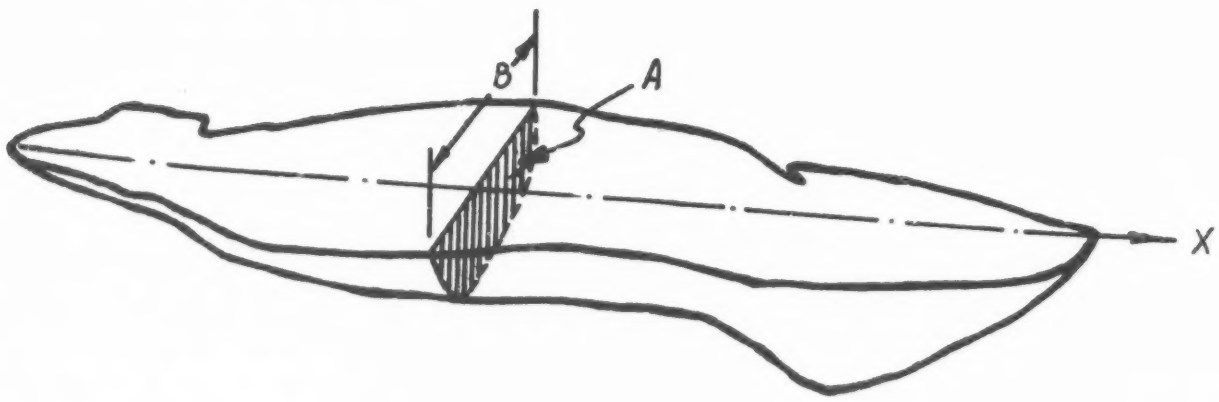
$$\eta = \eta_m \cos \frac{n\pi x}{\ell} \cos \frac{2\pi t}{T_n} \tag{8-4}$$

$$T_n = \frac{2\ell}{n\sqrt{gH}} \tag{8-5}$$

where ℓ denotes the length of the basin, n represents the number of nodal points, U_m is the maximum horizontal velocity, and η_m is the maximum vertical displacement of the free surface. Eq. (8-5) is the well-known



Rectangular Basin



Variable Geometry Basin

Fig. 31 Definition Sketch for Mass Oscillation in Lake Basins

Merian's formula for calculating the period of the n^{th} mode of the free oscillation. For a uninodal seiche, T is the time required for a shallow water wave to travel from one end of the basin to the other and back again. Merian's formula is a very useful first approximation in the analysis of seiches in basins of simple geometry.

The one dimensional equations of motion and continuity can be written taking into account topographical variations as (Hutchinson, 1957).

$$\frac{\partial \bar{u}}{\partial t} = - g \frac{\partial \eta}{\partial x} \tag{8-6}$$

$$\frac{\partial (A\bar{u})}{\partial x} + B \frac{\partial \eta}{\partial t} = 0 \tag{8-7}$$

where $\bar{u}$ is the average velocity component in the x-direction, A equals the cross sectional area of the basin, and B denotes the top width of the basin (see Fig. 31).

Equations (8-6) and (8-7) can be solved numerically when there is knowledge of A and B. (Platzman and Rao, 1964; Rockwell, 1966; Stewart and Rumer, 1970). The accuracy of such numerical calculations increases with the number of sections chosen (i.e., so that all variations in A and B are accounted for). However, where the basin geometry is conducive to significant transverse motion, errors will ensue. Utilizing the method of Stewart and Rumer (1970), the following results were obtained for the Lake Erie basin and compared to earlier computations and observations.

Table II. Lake Erie Periods

Mode	Observed Platzman (1965)	Merian Eq.(8-5)	Platzman and Rao (1964)	Calculated Results, Robson (1966) 17 Sections	53 Sections	65 Sections
1	14.1 hrs.	16.8 hrs.	14.08 hrs.	13.42 hrs.	13.70 hrs.	13.72 hrs.
2	9.2	8.4	8.92	8.28	8.63	8.61
3	6.0	5.6	5.70	5.65	5.67	5.63
4	4.1	4.2	4.11	4.45	4.24	4.11
5	-	3.36	3.69	3.75	3.69	3.62

Based on theoretical results, Platzman and Rao (1964) concluded that the earth's rotation had an effect of no more than 1.3% on the periods cited above.

VIII.2.2 Experimental Results

The results of experiments on seiches in the Lake Erie model have been reported by Rumer and Robson (1968). The response of the water level in the western basin of the model provided relatively smooth curves typical of simple harmonic motion. The seiche period observed in the western basin as determined experimentally, was 5.67 seconds which corresponds to 14.1 hr. in the prototype. The standard deviation of the many data calculated to be 0.045 sec or 6.7 min in prototype time. This agrees almost exactly with the periods cited in Table II.

Based on 71 observations, the decay modulus for the western basin of the physical model has been calculated to be 0.52. The decay modulus is defined as the natural logarithm of the ratio of succeeding wave heights spaced one period apart in time.

Response curves obtained from experiments in the central and eastern basin exhibited more complex wave forms. These results have not been analyzed.

The observed circulation pattern in the model during seiching reveals large eddies or gyres being formed in the channels separating the western basin from the central basin. These gyres grow to diameters of the order of 10 miles with rotational velocities of the same magnitude as the maximum longitudinal velocities of the seiche itself. They are formed as the water mass leaves the western basin and are swept out into the central basin where they decay. An aerial photographic mosaic of the western basin taken at 11:00 a.m. on 3 April, 1965 suggests the initial formation of such a gyre being formed off of Pelee Point. Analysis of water level records provided by the U.S. Lake Survey Ice and Snow Project and the Buffalo District Corps of Engineers confirmed the existence of a longitudinal seiche during that period which very likely accounts for the gyre formation.

Significant increases in current speeds over the steady state circulation velocities are observed during seiches in the model. This increase is enough to mask the steady-state flow pattern. Using the calculations of Platzman and Rao (1963), the rate of volume transport between the western and central basin, for a 1 foot amplitude at Buffalo, is approximately 5×10^6 cfs or 30 times the normal throughflow of 175,000 cfs. This movement of water during seiching occurs across the whole lake cross section and not just through preferred channels as does the throughflow. Between the central and eastern basins the volume rate of transport is 34 times as great as the throughflow.

The agreement between calculated and experimentally observed periods for the first mode of oscillation in Lake Erie served as a positive contribution to increasing the level of confidence in the model operation.

VIII.3 Free Damped Oscillations

In the absence of wind, the potential and kinetic energy of a free oscillation will be dissipated by the frictional resistance of the basin boundaries and by internal viscous dissipation. In a shallow basin, such as Lake Erie, this frictional damping should be significant. The development of a mathematical model capable of predicting the viscous decay of free oscillations in basins of arbitrary geometry seemed desirable because of the possibility of using such a mathematical model in conjunction with a physical model for scaling friction. If reliable prototype data were available on the transient decay of seiches, the proposed method seemed to be ideally suited to calibrating the frictional effects in models of large lakes. During seiching, the effects of the earth's rotation can reasonably be neglected in most instances. The remaining forces influencing the motion are gravity and friction.

This proposition motivated a study (still underway) of frictional damping in large lakes. The study began with an experimental investigation of the decay of free oscillations in the rectangular basin of constant depth. The bottom roughness of the model basin was artificially roughened providing as one experimental variable the relative roughness ratio, k/H where k is the average height of the roughness elements. The bottom was artificially roughened with coarse sand and cylindrical disks. Smooth bottoms of plexiglass and painted plywood were also investigated. Experimental results from the Lake Erie model were also available. The bottom of the Lake Erie model has a painted epoxy surface.

The theoretical study has included a one dimensional analysis of frictional damping in rectangular basins of variable relative roughness ratios. A two-dimensional laminar flow approach was also explored.

The basic equations for the former approach are (Defant, 1961)

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + g\frac{\partial \eta}{\partial x} = -\frac{\tau_o}{\rho H} \tag{8-8}$$

$$\frac{\partial}{\partial x}\left[(H + \eta)\bar{u}\right] + \frac{\partial \eta}{\partial t} = 0 \tag{8-9}$$

where τ_o is the shear stress on the bottom. The other terms are as defined earlier. For shallow water waves, these equations can be simplified to

$$\frac{\partial \bar{u}}{\partial t} + g\frac{\partial \eta}{\partial x} + \frac{\tau_o}{\rho H} = 0 \tag{8-10}$$

$$H\frac{\partial \bar{u}}{\partial x} + \frac{\partial \eta}{\partial t} = 0 \tag{8-11}$$

The solution of equations (8-10) and (8-11) is dependent upon the assumed form for τ_o. One solution has been obtained assuming that the shear stress, τ_o, is linearly related to the horizontal velocity, $\bar{u}$. A second approximate solution was obtained assuming that $\tau_o = \frac{f}{8}|\bar{u}|\bar{u}$ where f is the so-called friction coefficient.

Work is still underway on this aspect of the Lake Erie project. Attention is being given to extension of the analytical work to include arbitrary geometry. More experiments are being planned so that appropriate comparison with the analytical models can be made. Ultimately, it is hoped that a reliable and practical technique for scaling friction will become available from this study of the decay of seiches.

VIII.4 Dispersion During Seiching

The mixing and dilution of wastes discharged into lakes depends upon the wind and bottom shear-generated turbulence as well as convection due to the large scale circulation pattern. Although transient in effect, it appears that the phenomenon of seiching can be a significant factor

in the mixing of wastes. Already cited are the large volume transports during seiching calculated for Lake Erie (Platzman and Rao, 1964). These volume transports can be on the order of 50 times greater than the steady state transport due to throughflow.

During seiching the motion in a lake basin is essentially one dimensional, horizontal, and periodic. It will often be the predominate motion when it occurs and will mask the other currents normally present. The periodic nature of this flow phenomenon causes it to have certain similarities with the mixing problem in estuaries. The estuary problem has received extensive study and the results of these studies should be carefully studied in relationship to the dispersion of wastes during seiching.

The general three dimensional equation describing the transport of a pollutant in a fluid has been given earlier as eq.(7-1). A simplified one-dimensional mass balance equation can be written as, assuming the principal direction of flow is along the x axis, (Holley and Harleman, 1965)

$$\frac{\partial \bar{s}}{\partial t} + \bar{u} \frac{\partial \bar{s}}{\partial x} = \frac{1}{A} \frac{\partial}{\partial x} (\bar{E}_x A \frac{\partial \bar{s}}{\partial x}) \qquad (8\text{-}12)$$

where $\bar{s}$ is the average concentration of the pollutant at a cross section, $\bar{u}$ is the instantaneous average velocity at a cross section, and $\bar{E}_x$ is the cross sectional average of the dispersion coefficient. The dispersion coefficient takes into account the presence of spatial variations in the actual velocity profile from the average value, $\bar{u}$.

Fischer (1967) has pointed out that eq.(8-12) can not be assumed valid during an initial or convective period in which the movement of the pollutant is still dependent on initial convective velocities. He

states that eq.(8-12) is probably not applicable to wide streams. Based on his analysis, it is unlikely that a one-dimensional approach can be utilized in the study of pollutant mixing in large lakes even though during seiching, the convective velocity may be approximated by a unidirectional flow.

Thus, it appears that a more appropriate mass balance equation would be

$$\frac{\partial s}{\partial t} + \bar{u}\frac{\partial s}{\partial x} = \frac{\partial}{\partial x}\left(E_x \frac{\partial s}{\partial x}\right) + \frac{\partial}{\partial y}\left(E_y \frac{\partial s}{\partial y}\right) + \frac{\partial}{\partial z}\left(E_z \frac{\partial s}{\partial z}\right) \qquad (8\text{-}13)$$

The velocity, $\bar{u}$, may be assumed to vary (for the uninodal seiche) as a periodic exponentially decaying function of time. It will also be equal to zero at both ends of the basin and have its maximum value at the node of the standing wave pattern. For the rectangular basin shown in Fig. 31 the velocity $\bar{u}$ can be represented in a simplistic way as

$$\bar{u} = U_m e^{-\alpha t} \sin\frac{\pi x}{\ell} \sin\frac{2\pi}{T} t \qquad (8\text{-}14)$$

where U_m is the maximum amplitude of the velocity $\bar{u}$, T is the period of the uninodal seiche, and α is a decay modulus. Eq. (8-14) prescribes the maximum amplitude of the velocity as a decaying function of time.

The study of the spread of pollutants utilizing the basic approach suggested by eqs. (8-13) and (8-14) is presently underway. For a variable geometry basin the velocity, $\bar{u}$, can be approximated using the results of numerical computations obtained from the solution of the free undamped eqs.(8-6) and (8-7) and modifying them in accordance with separately obtained information on the decay modulus, α, for the particular basin in question.

The precedure used in the study of estuary mixing of time averaging the terms in the mass balance equation over a tidal period results in

the convective velocity representing only the steady-state throughflow. The mixing associated with the convection due to the periodic velocity is then incorporated into the dispersion term. This approach is probably not appropriate for lake basins during seiching because of the relatively few number of cycles involved until the water mass returns to static equilibrium.

A more complete report of this part of the project will be published at a later date.

IX. SUMMARY

The use of rotating hydraulic models of large lakes appears to be a practical and useful technique for studying lake circulation. The use of these models enables the kind of controlled experiments necessary for comparison with results obtained from mathematical models. The experience of this investigation has shown that rotating models can be operated and that the reproducibility of results is good.

Based on the experimental and analytical results of this study, the following summarizing statements are made regarding the dynamic behavior of Lake Erie.

(1) In the absence of wind, the throughflow caused by the inflow of the Detroit River and the outflow of the Niagara River generates significant currents and easily observable patterns of circulation. Because of the concentration of these currents, observed velocities are often greater than an average velocity obtained by dividing the total discharge by the cross sectional area. The observed steady state pattern of circulation shows a large counterclockwise flow in the western basin. The bulk of this flow exits the western basin through Pelee Passage and then flows south joining the other portions of the flow that have exited the western

basin through the islands. This collected flow is diverted eastward by the southern shore where it maintains its identity all the way to the Niagara River. The central basin has a very slow westward drift that feeds a stronger eastward current along the north shore. The surface currents in the eastern basin flow eastward and converge on the Niagara River. This observed circulation pattern should simulate the prototype circulation pattern whenever wind stress can be considered unimportant (e.g., after several days of calm or under ice cover). The "inertial theory" for oceanic circulation predicts some of the important features of this observed circulation pattern in the model.

(2) The presence of a westerly wind significantly alters the circulation pattern of Lake Erie as observed in the absence of wind. The homogeneous lake develops separate surface and subsurface circulation patterns under the action of a westerly wind. These patterns are coupled by vertical motions (sinking and upwelling). The surface pattern is dominated by a southeasterly flow with a strong eastward south shore current. The subsurface pattern is more complex and marked by stable gyre formations and transverse flows north and south across the lake.

(3) A procedure for relating model wind speeds to prototype wind speeds based on similarity between wind set-up in model and prototype has been presented.

(4) The period for the first mode of mass oscillation in the model lake is in good agreement with calculated and observed periods. The development of a suitable mathematical model for frictional damping of seiches promises to be useful in the scaling of friction in hydraulic models of lakes.

The observed translatory motion during seiching completely masks the steady-state circulation pattern due to throughflow. Mass transport between basins during seiching is approximately 50 times the mass transport due to the throughflow.

(5) Observed detention periods show significant short circuiting for the homogeneous lake with no wind stress. In the presence of a westerly wind this detention period is reduced even further. An empirical approach has been developed utilizing a transition probability matrix for predicting the spread of pollutants in the western basin due to the Detroit River inflow. This approach predicts pollutant distributions in good agreement with experimental observations.

(6) Only limited correspondence can be achieved in modeling dispersion in vertically distorted Froude models of very large bodies of water.

The following aspects of this study are still underway and will be reported on in subsequent publications.

(1) Continued development of a mathematical model for lake circulation including hydraulic throughflow, wind stress, and vertical and horizontal diffusion of momentum.

(2) An investigation of the frictional damping of seiches with the objective of developing a method for scaling friction in hydraulic models of large lakes. This investigation is both experimental and analytical.

(3) An analysis of the mixing of pollutants during seiching.

(4) A study of the transient response of a lake surface to a suddenly imposed wind stress.

One final recommendation resulting from this investigation is that, whenever extensive field studies are being contemplated for a large lake, an operating scale model should be constructed to aid in the planning of the field study and to aid in the interpretation of the field data collected.

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